MATHEMATICS

Time : 3 Hrs. Max. Marks 85

Special Instructions :

1. You must write Question Paper Series in the circle at top left side of title page of your answer book.
2. While answering your questions you must indicate on your answer book the same question number as appears in your question paper.
3. Do not leave blank page(s) in your answer book.
4. Q.No. 1 to 10 multiple choice questions are of 1 mark each. Questions 11 to 14 are of 2 marks each, Q.Nos. 15 to 26 are of 3½ marks each and Q.Nos. 27 to 31 are of 5 marks each.
5. All questions are compulsory.
6. Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.
7. Use of calculator is not permitted, however, you ask for logarithmic tables, if required from the superintendent of examinations.
8. Try to answer the questions in serial order as far as possible.

Q1. If $\sin^{-1} x = y$ then

(a) $0 \leqslant y \leqslant \pi$ (b) $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Q2. $A = [a_{ij}]_{m \times n}$ is a square matrix if

(a) $m < n$ (b) $m > n$ (c) $m = n$ (d) None

Q3. Derivative of $(ax + b)^n$ is :

(a) $\frac{n(ax+b)^{n-1}}{a}$ (b) $\frac{(ax+b)^{n-1}}{a}$ (c) $n(ax+b)^{n-1}$ (d) $na(ax+b)^{n-1}$

Q4. The rate of change of the area of a circle with respect to its radius r at r = 6cm is:

(a) 10π (b) 12π (c) 8π (d) 11π

Q5. $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2}$ equals :

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

Q6. The order of differential equation
$2x^2y^{11} - 3y^1 + y = 0$ is
(a) 2 (b) 1 (c) 0 (d) not defined

Q7. If θ is angle between $\vec{a}$ and $\vec{b}$, then $|\vec{a} . \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to :
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

Q8. If $\vec{a}$ is a non zero vector of magnitude 'a' and λ a non zero scalar, then $\lambda\vec{a}$ is a unit vector if :
(a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$

Q9. The planes : $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are :
(a) Perpendicular (b) Parallel
(c) Intersect y-axis (d) passes through $\left(0, 0, \frac{\Sigma}{4}\right)$

Q10. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then P (A/B) is
(a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

Q11. Find x and y if $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$

OR

By using elementary operations, find the inverse of matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

Q12. Examine the function : $f(x) = \frac{1}{x - 5}$ for continuity :

Q13. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2)

Q14. Solve the differential equation :
$y' + y = 1$

Q15. Find gof and fog if
$f(x) = |x|$ and $g(x) = |5x - 2|$

Q16. Show that :
$\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x, \quad -\frac{1}{\sqrt{2}} \leqslant x \leqslant \frac{1}{\sqrt{2}}$

OR

Prove that :

$$\tan^{-1} x + \tan^{-1} \frac{2x}{1 - x^2} = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \quad |x| < \frac{1}{\sqrt{3}}$$

Q17. By using properties of determinant prove that :

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Q18. Is the function defined by :

$$f(x) = \begin{cases} x + 5, & \text{if } x \leqslant 1 \\ x - 5, & \text{if } x > 1 \end{cases}$$

a continuous function ?

OR

Differentiate $\sin\left(\cos(x^2)\right)$ w.r.t. x

Q19. Integrate $\int x \sec^2 x \, dx$

Q20. Find $\int_0^2 (x^2 + 1)\, dx$ as the limit of a sum.

Q21. By using properties of definite integrals, evaluate :

$$\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$$

Q22. Solve the differential equation :

$(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$

OR

Solve the differential equation

$(x^2 + xy)\, dy = (x^2 + y^2)\, dx$

Q23. Find the area of a triangle having the points A (1, 1, 1), B(1, 2, 3) and C (2, 3, 1) as its vertices.

Q24. Show that the lines

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \quad \text{and} \quad \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$$

are coplanar

Q25. Find the probability distribution of

(i) Number of heads in two tosses of a coin.

(ii) Number of tails in the simultaneous tosses of three coins.

(iii) Number of heads in four tosses of a coin.

Q26. Find the probability of getting 5 exactly twice in 7 throws of a dice

OR

Let E and F be events with $P(E) = \frac{3}{5}$ $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$.

Are E and F independent ?

Q27. Solve the following system of equations by matrix method :

$3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$

Q28. Find the local maxima and local minima if any of the function :

$f(x) = x^3 - 6x^2 + 9x + 15$

OR

A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of two pieces, so that the combined area of the square and the circle is minimum.

Q29. Find the area of the region in the first quadrant enclosed by the x-axis, the line y=x and the circle $x^2 + y^2 = 32$

OR

Find the area under the given curves and the given lines.

$y = x^2$, $x = 1$, $x = 2$ and x – axis

Q30. Find the shortest distance between the lines

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

OR

Find the equations of the planes that passes through three points

(1, 1, –1), (6, 4, –5), (–4, –2, 3)

Q31. Minimise $Z = -3x + 4y$

subject to $x + 2y \leqslant 8$, $3x + 2y \leqslant 12$, $x \geqslant 0$, $y \geqslant 0$