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UPSC NDA Exam 2010-I Mathematics Solved Paper

Mathematics

1. What is the number of ways of arranging the letters of the word 'BANANA' so that no two N's appear together ?
(a) 40 (b) 60
(c) 80 (d) 100

2. Consider the equation $(x-p)(x-6)+1=0$ having integral coefficients. If the equation has integral roots, then what values can p have ?
(a) 4 or 8 (b) 5 or 10
(c) 6 or 12 (d) 3 or 6

3. What is the equivalent binary number of the decimal number 13.625 ?
(a) 1101.111 (b) 1111.101
(c) 1101.101 (d) 1111.111

4. What is the value of $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{200}+\left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200}+1$?
(a) -1 (b) 0
(c) 1 (d) 2

5. The order of a set A is 3 and that of a set B is 2. What is the number of relations from A to B ?
(a) 4 (b) 6
(c) 32 (d) 64

6. What is the value of $\frac{\log_{\sqrt{\alpha\beta}}(H)}{\log_{\sqrt{\alpha\beta\gamma}}(H)}$?
(a) $\log_{\alpha\beta}(\alpha)$ (b) $\log_{\alpha\beta\gamma}(\alpha\beta)$
(c) $\log_{\alpha\beta}(\alpha\beta\gamma)$ (d) $\log_{\alpha\beta}(\beta)$

7. The 59th term of an AP is 449 and the 449th term is 59. Which term is equal to 0 (zero) ?
(a) 501st term (b) 502nd term
(c) 508th term (d) 509th term

8. For a set A, consider the following statements
1. $A \cup P(A) = P(A)$
2. $\{A\} \cap P(A) = A$
3. $P(A) - \{A\} = P(A)$
where P denotes power set.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) 3 only (d) 1, 2 and 3

9. If the AM and HM of two numbers are 27 and 12 respectively, then what is their GM equal to ?
(a) 12 (b) 18
(c) 24 (d) 27

10. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then what is the value of $(A+B)$?
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

11. If (4, 0) and (– 4, 0) are the foci of an ellipse and the semi-minor axis is 3, then the ellipse passes through which one of the following points ?
(a) (2, 0) (b) (0, 5)
(c) (0, 0) (d) (5, 0)

12. Under what condition do the planes $bx - ay = n$, $cy - bz = l$, $az - cx = m$ intersect in a line ?
(a) $a + b + c = 0$
(b) $a = b = c$
(c) $al + bm + cn = 0$
(d) $l + m + n = 0$

13. What is the maximum point on the curve $x = e^x y$?
(a) $(1, e)$ (b) $(1, e^{-1})$
(c) $(e, 1)$ (d) $(e^{-1}, 1)$

14. The function $f(x) = e^x$, $x \in R$ is
(a) onto but not one-one
(b) one-one onto
(c) one-one but not onto
(d) neither one-one nor onto

15. If $y = \sin^{-1}\left(\frac{4x}{1+4x^2}\right)$, then what is $\frac{dy}{dx}$ equal to ?
(a) $\frac{1}{1+4x^2}$ (b) $-\frac{1}{1+4x^2}$
(c) $\frac{4}{1+4x^2}$ (d) $\frac{4x}{1+4x^2}$

Directions *For the next 3 (three) questions to follow :*
The table below gives an incomplete frequency distribution with two missing frequencies f_1 and f_2.

Value of x	Frequency
0	f_1
1	f_2
2	4
3	4
4	3

The total frequency is 18 and the arithmetic mean of x is 2.

16. What is the value of f_2 ?
(a) 4 (b) 3
(c) 2 (d) 1

17. What is the standard deviation ?
(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{3}$
(c) $\frac{4}{3}$ (d) $\frac{16}{9}$

18. What is the coefficient of variance ?
(a) $\frac{200}{3}$ (b) $\frac{50\sqrt{5}}{9}$
(c) $\frac{600}{\sqrt{5}}$ (d) 150

19. What is the sum of all natural numbers between 200 and 400 which are divisible by 7 ?
(a) 6729 (b) 8712
(c) 8729 (d) 9276

20. The mean and variance of a binomial distribution are 8 and 4 respectively. What is $P(X=1)$ equal to ?
(a) $\frac{1}{2^{12}}$ (b) $\frac{1}{2^8}$
(c) $\frac{1}{2^6}$ (d) $\frac{1}{2^4}$

21. What is $\int e^{\ln x} \sin x \, dx$ equal to ?

(a) $e^{\ln x}(\sin x - \cos x) + c$ (b) $(\sin x - x \cos x) + c$

(c) $(x \sin x + \cos x) + c$ (d) $(\sin x + x \cos x) - c$

Where c is a constant of integration.

22. An observed event B can occur after one of the three events A_1, A_2, A_3. If

$$P(A_1) = P(A_2) = 0.4,\ P(A_3) = 0.2$$

and $P(B/A_1) = 0.25, P(B/A_2) = 0.4,$

$P(B/A_3) = 0.125$, what is the probability of A_1 after observing B ?

(a) $\frac{1}{3}$ (b) $\frac{6}{19}$

(c) $\frac{20}{57}$ (d) $\frac{2}{5}$

23. What is $\int \frac{x^4+1}{x^2+1} dx$ equal to ?

(a) $\frac{x^3}{3} - x + 4\tan^{-1} x + c$

(b) $\frac{x^3}{3} + x + 4\tan^{-1} x + c$

(c) $\frac{x^3}{3} - x + 2\tan^{-1} x + c$

(d) $\frac{x^3}{3} - x - 4\tan^{-1} x + c$

where c is a constant of integration.

24. What is the value of $\lim_{x \to \infty} \left(\frac{x+6}{x+1}\right)^{x+4}$

(a) e (b) e^2

(c) e^4 (d) e^5

25. What is the area of the triangle with vertices $(0, 2, 2), (2, 0, -1)$ and $(3, 4, 0)$?

(a) $\frac{15}{2}$ sq unit (b) 15 sq unit

(c) $\frac{7}{2}$ sq unit (d) 7 sq unit

26. If $\frac{1}{2-\sqrt{-2}}$ is one of the roots of $ax^2 + bx + c = 0$ where, a, b, c are real, then what are the values of a, b, c respectively ?

(a) 6, – 4, 1 (b) 4, 6, – 1

(c) 3, – 2, 1 (d) 6, 4, 1

27. If ω is a complex cube root of unity and $x = \omega^2 - \omega - 2$, then what is the value of $x^2 + 4x + 7$?

(a) – 2 (b) – 1

(c) 0 (d) 1

28. If $\cos x \neq -1$, then what is $\frac{\sin x}{1 + \cos x}$ equal to ?

(a) $-\cot\frac{x}{2}$

(b) $\cot\frac{x}{2}$

(c) $\tan\frac{x}{2}$

(d) $-\tan\frac{x}{2}$

29. The angle of elevation of the top of a flag post from a point 5 m away from its base is 75°. What is the approximate height of the flag post ?

(a) 15 m (b) 17 m

(c) 19 m (d) 21 m

30. If $A = P(\{1, 2\})$ where P denotes the power set, then which one of the following is correct ?

(a) $\{1, 2\} \subset A$ (b) $1 \in A$

(c) $\phi \notin A$ (d) $\{1, 2\} \in A$

31. Let $O(0, 0, 0)$, $P(3, 4, 5)$, $Q(m, n, r)$ and $R(1, 1, 1)$ be the vertices of a parallelogram taken in order. What is the value of $m + n + r$?

(a) 6 (b) 12

(c) 15 (d) More than 15

32. What is the solution of the differential equation $3e^x \tan y \, dx + (1 + e^x)\sec^2 y \, dy = 0$?

(a) $(1 + e^x)\tan y = c$

(b) $(1 + e^x)^3 \tan y = c$

(c) $(1 + e^x)^2 \tan y = c$

(d) $(1 + e^x)\sec^2 y = c$

where c is a constant of integration.

33. What is the locus of points, the difference of whose distances from two points being constant ?

(a) Pair of straight lines

(b) An ellipse

(c) A hyperbola

(d) A parabola

34. What is the differential equation for $y^2 = 4a(x - a)$?

(a) $yy' - 2xyy' + y^2 = 0$

(b) $yy'(yy' + 2x) + y^2 = 0$

(c) $yy'(yy' - 2x) + y^2 = 0$

(d) $yy' - 2xyy' + y = 0$

35. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, what is the angle between $-5\vec{a}$ and $6\vec{b}$?

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$

(c) $\frac{2\pi}{5}$ (d) $\frac{3\pi}{7}$

36. What is the degree of the differential equation

$$\frac{d^2y}{dx^2} - \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0 ?$$

(a) 1 (b) 2

(c) 3 (d) 6

37. If $\int x^2 \ln x \, dx = \frac{x^3}{m}\ln x + \frac{x^3}{n} + c$, then what are the values of m and n respectively ?

(a) $\frac{1}{3}, -\frac{1}{9}$ (b) 3, – 9

(c) 3, 9 (d) 3, 3

where c is a constant of integration.

38. What is the principle value of $\text{cosec}^{-1}(-\sqrt{2})$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $-\frac{\pi}{4}$ (d) 0

39. If $f: R \to R, g: R \to R$ and $g(x) = x + 3$ and $(fog)(x) = (x + 3)^2$, then what is the value of $f(-3)$?

(a) -9 (b) 0
(c) 9 (d) 3

40. What is the value of $\lim_{x \to 1} \frac{(x-1)^2}{|x-1|}$?

(a) 0
(b) 1
(c) -1
(d) The limit does not exist

41. A balloon is pumped at the rate of 4 cm^3 per second. What is the rate at which its surface area increases when its radius is 4 cm?

(a) 1 cm^2/s (b) 2 cm^2/s
(c) 3 cm^2/s (d) 4 cm^2/s

42. What is the value of $\frac{1 + \tan 15^\circ}{1 - \tan 15^\circ}$?

(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

43. If $f(x) = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then which one of the following is correct?

(a) $k < 3$ (b) $k \le 3$
(c) $k > 3$ (d) $k \ge 3$

44. If $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$, then what is the value of x?

(a) 1 (b) 7
(c) 13 (d) 17

45. If α, β are the roots of the quadratic equation $x^2 - x + 1 = 0$, then which one of the following is correct?

(a) $(\alpha^4 - \beta^4)$ is real
(b) $2(\alpha^5 + \beta^5) = (\alpha\beta)^5$
(c) $(\alpha^6 - \beta^6) = 0$
(d) $(\alpha^8 + \beta^8) = (\alpha\beta)^8$

46. If $\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 2$, then what is the value of the determinant

$$\begin{vmatrix} 6a & 3b & 15c \\ 2l & m & 5n \\ 2p & q & 5r \end{vmatrix}?$$

(a) 10 (b) 20
(c) 40 (d) 60

47. Let X be the set of all graduates in India. Elements x and y in X are said to be related if they are graduates of the same university. Which one of the following statements is correct?

(a) Relation is symmetric and transitive only
(b) Relation is reflexive and transitive only
(c) Relation is reflexive and symmetric only
(d) Relation is reflexive symmetric and transitive

48. If $x^2 + y^2 = 1$, then what is $\frac{1 + x + iy}{1 + x - iy}$ equal to?

(a) $x - iy$
(b) $x + iy$
(c) $2x$
(d) $-2iy$

49. Consider the following statements

1. For any three vectors $\vec{a}, \vec{b}, \vec{c}$;

$$\vec{a} \cdot \{(\vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c})\} = 0$$

2. For any three coplanar unit vectors

$$\vec{d}, \vec{e}, \vec{f}; (\vec{d} \times \vec{e}) \cdot \vec{f} = 1$$

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Directions *For the next 3 (three) questions to follow consider the following lists : Each item under List I is associated with one or more items under List II.*

	List I (Function)		List II (Property)
A.	$\sin x$	1.	Periodic function
B.	$\cos x$	2.	Non-periodic function
C.	$\tan x$	3.	Continuous at every point on $(-\infty, \infty)$
		4.	Discontinuous function
		5.	Differentiable at every point on $(-\infty, \infty)$
		6.	Not differentiable at every point on $(-\infty, \infty)$
		7.	has period π
		8.	has period 2π
		9.	increases on $\left(0, \frac{\pi}{2}\right)$
		10.	decreases on $\left(0, \frac{\pi}{2}\right)$
		11.	increases on $\left(\frac{\pi}{2}, \pi\right)$
		12.	decreases on $\left(\frac{\pi}{2}, \pi\right)$

50. A is associated with

(a) 1, 3, 5, 8, 9, 12 (b) 2, 4, 6, 8, 10, 11
(c) 1, 3, 5, 7, 10, 11 (d) None of these

51. B is associated with

(a) 2, 3, 5, 8, 9, 12 (b) 1, 3, 5, 8, 10, 12
(c) 1, 3, 5, 8, 9, 12 (d) None of the above

52. C is associated with

(a) 1, 4, 6, 7, 9, 11 (b) 2, 4, 6, 8, 9
(c) 1, 4, 6, 7, 9 (d) None of these

53. If p and q are positive integers, then which one of the following equations has $p - \sqrt{q}$ as one of its roots?

(a) $x^2 - 2px - (p^2 - q) = 0$
(b) $x^2 - 2px + (p^2 - q) = 0$
(c) $x^2 + 2px - (p^2 - q) = 0$
(d) $x^2 + 2px + (p^2 - q) = 0$

54. Given two squares of sides x and y such that $y = x + x^2$. What is the rate of change of area of the second square with respect to the area of the first square ?
(a) $1 + 3x + 2x^2$
(b) $1 + 2x + 3x^2$
(c) $1 - 2x + 3x^2$
(d) $1 - 2x - 3x^2$

55. The planes $px + 2y + 2z - 3 = 0$ and $2x - y + z + 2 = 0$ intersect at an angle $\frac{\pi}{4}$. What is the value of p^2 ?
(a) 24 (b) 12
(c) 6 (d) 3

56. The growth of a quantity $N(t)$ at any instant t is given by $\frac{dN(t)}{dt} = \alpha N(t)$. Given that $N(t) = ce^{kt}$, c is a constant. What is the value of α ?
(a) c (b) k
(c) $c + k$ (d) $c - k$

57. A circle is drawn with the two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the end of the diameter. What is the equation of the circle ?
(a) $x^2 + y^2 = a^2 + b^2$
(b) $x^2 + y^2 = a^2 - b^2$
(c) $x^2 + y^2 = 2(a^2 + b^2)$
(d) $x^2 + y^2 = 2(a^2 - b^2)$

58. What is the image of the point (1, 2) on the line $3x + 4y - 1 = 0$?
(a) $\left(-\frac{7}{5}, -\frac{6}{5}\right)$ (b) $\left(\frac{7}{8}, \frac{1}{2}\right)$
(c) $\left(\frac{7}{8}, -\frac{1}{2}\right)$ (d) $\left(-\frac{7}{5}, \frac{1}{2}\right)$

59. If the product of the roots of the equation $x^2 - 5x + k = 15$ is -3, then what is the value of k ?
(a) 12 (b) 15
(c) 16 (d) 18

60. Consider the following statements
1. Every function has a primitive.
2. A primitive of a function is unique.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

61. What is the value of $\sqrt{3}$ cosec $20° - \sec 20°$?
(a) 1/4 (b) 4
(c) 2 (d) 1

62. The probability distribution of random variable X with two missing probabilities p_1 and p_2 is given below

X	$P(X)$
1	k
2	p_1
3	$4k$
4	p_2
5	$2k$

It is further given that $P(X \le 2) = 0.25$ and $P(X \ge 4) = 0.35$.
Consider the following statements
1. $p_1 = p_2$
2. $p_1 + p_2 = P(X = 3)$
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

63. If angles A, B, C are in AP, then what is $\sin A + 2\sin B + \sin C$ equal to ?
(a) $4\sin B \cos^2\left(\frac{A-C}{2}\right)$ (b) $4\sin B \cos^2\left(\frac{A-C}{4}\right)$
(c) $4\sin(2B)\cos^2\left(\frac{A-C}{2}\right)$ (d) $4\sin(2B)\cos^2\left(\frac{A-C}{4}\right)$

64. **Statement I** If $-1 \le x < 0$, then $\cos(\sin^{-1} x) = -\sqrt{1 - x^2}$
Statement II If $-1 \le x < 0$, then $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$
Which one of the following is correct in respect of the above statements ?
(a) Both statements I and II are independently correct and statement II is the correct explanation of statement I
(b) Both statements I and II are independently correct but statement II is not the correct explanation of statement I
(c) Statement I is correct but statement II is false
(d) Statement I is false but statement II is correct.

65. **Statement I** $y = -\tan^{-1}(x^{-1}) + 1$ is an increasing function of x.
Statement II $\frac{dy}{dx}$ is positive for all values of x.
Which one of the following is correct in respect of the above statements ?
(a) Both statements I and II are independently correct and statement II is the correct explanation of statement I
(b) Both statements I and II are independently correct but statement II is not the correct explanation of statement I
(c) Statement I is correct but statement II is false
(d) Statement I is false but statement II is correct

66. Consider the following statements in respect of circles $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 1$
1. The radius of the first circle is twice that of the second circle.
2. Both the circles pass through the origin.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

67. Let a, b, c be in AP.
Consider the following statements
1. $\frac{1}{ab}, \frac{1}{ca}, \frac{1}{bc}$ are in AP.
2. $\frac{1}{\sqrt{b} + \sqrt{c}}, \frac{1}{\sqrt{c} + \sqrt{a}}, \frac{1}{\sqrt{a} + \sqrt{b}}$ are in AP.

Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

68. What is the differentiation of $\log_x x$ with respect to $\ln x$?
(a) 0 (b) 1
(c) $1/x$ (d) x

69. What is $\tan\left(7\frac{1}{2}\right)^\circ$ equal to ?
(a) $\sqrt{6}+\sqrt{3}-\sqrt{2}+2$ (b) $\sqrt{6}+\sqrt{3}+\sqrt{2}+2$
(c) $\sqrt{6}-\sqrt{3}+\sqrt{2}-2$ (d) $\sqrt{6}+\sqrt{3}+\sqrt{2}-2$

70. What is the value of $\frac{\cos 15^\circ + \cos 45^\circ}{\cos^3 15^\circ + \cos^3 45^\circ}$?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) None of these

Directions *For the next 3 (three) questions to follow :* *The vertices of a cube are* (0, 0, 0), (2, 0, 0), (0, 2, 0), (0, 0, 2), (2, 2, 0), (2, 0, 2), (0, 2, 2), (2, 2, 2) *respectively.*

71. What is the angle between any two diagonals of the cube ?
(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/3)$
(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\cos^{-1}(2/\sqrt{3})$

72. What is the angle between one of the edges of the cube and the diagonal of the cube intersecting the edge of the cube ?
(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1}(1/3)$
(c) $\cos^{-1}(1/\sqrt{3})$ (d) $\cos^{-1}(2/\sqrt{3})$

73. What is the angle between the diagonal of one of the faces of the cube and the diagonal of the cube intersecting the diagonal of the face of the cube ?
(a) $\cos^{-1}(1/\sqrt{3})$ (b) $\cos^{-1}(2/\sqrt{3})$
(c) $\cos^{-1}(\sqrt{2/3})$ (d) $\cos^{-1}(\sqrt{2}/3)$

74. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them. If $(\vec{a}+\vec{b})$ is also the unit vectors, then what is the value of α ?
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

75. What is the value of
$$\frac{(0.101)_2^{(11)_2} + (0.011)_2^{(11)_2}}{(0.101)_2^{(10)_2} - (0.101)_2^{(01)_2}(0.011)_2^{(01)_2} + (0.011)_2^{(10)_2}}$$
(a) $(0.001)_2$ (b) $(0.01)_2$
(c) $(0.1)_2$ (d) $(1)_2$

76. If $A = \{a, b, c, d\}$, then what is the number of proper subsets of A ?
(a) 16 (b) 15
(c) 14 (d) 12

77. What is the number of three-digit odd numbers formed by using the digits 1, 2, 3, 4, 5, 6 if repetition of digits is allowed ?
(a) 60 (b) 108
(c) 120 (d) 216

78. Let $A = \begin{bmatrix} 5 & 6 & 1 \\ 2 & -1 & 5 \end{bmatrix}$. Let there exist a matrix B such that $AB = \begin{bmatrix} 35 & 49 \\ 29 & 13 \end{bmatrix}$. What is B equal to ?
(a) $\begin{bmatrix} 5 & 1 & 4 \\ 2 & 6 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 & 3 \\ 5 & 1 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 5 & 2 \\ 1 & 6 \\ 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 5 \\ 6 & 1 \\ 3 & 4 \end{bmatrix}$

79. Consider the following statements
1. The probability that there are 53 Sundays in a leap year is twice the probability that there are 53 Sundays in a non-leap year.
2. The probability that there are 5 Mondays in the month of March is thrice the probability that there are 5 Mondays in the month of April.

Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

80. Consider the following statements
1. If $A' = A$; then A is a singular matrix, where A' is the transpose of A.
2. If A is a square matrix such that $A^3 = I$, then A is non-singular.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

81. If p times the pth term of an AP is q times the qth term, then what is the $(p+q)$th term equal to ?
(a) $p+q$ (b) pq
(c) 1 (d) 0

82. A team of 8 players is to be chosen from a group of 12 players. Out of the eight players one is to be elected as captain and another an vice-captain. In how many ways can this is done ?
(a) 27720 (b) 13860
(c) 6930 (d) 495

83. In tossing three coins at a time, what is the probability of getting at most one head ?
(a) $\frac{3}{8}$ (b) $\frac{7}{8}$
(c) $\frac{1}{2}$ (d) $\frac{1}{8}$

84. What is the sum of the coefficients of all the terms in the expansion of $(45x - 49)^4$?
(a) –256 (b) –100
(c) 100 (d) 256

85. Two balls are selected from a box containing 2 blue and 7 red balls. What is the probability that at least one ball is blue ?
(a) $\frac{2}{9}$ (b) $\frac{7}{9}$
(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

86. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then which one of the following is correct ?

(a) $-3 < b < 3$ (b) $-2 < b < 2$
(c) $b > 2$ (d) $b < -2$

87. The probability of guessing a correct answer is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{2}{3}$, then what is x equal to ?
(a) 2 (b) 3
(c) 4 (d) 6

88. If the system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then which one of the following is correct ?
(a) $a = 2b$ (b) $b = 2a$
(c) $a = -2b$ (d) $b = -2a$

89. If p and q are the roots of the equation $x^2 - px + q = 0$, then what are the values of p and q respectively ?
(a) 1, 0 (b) 0, 1
(c) -2, 0 (d) -2, 1

90. Consider the following statements related to a variable X having a binomial distribution $b_X(n, p)$
1. If $p = \frac{1}{2}$, then the distribution is symmetrical.
2. p remaining constant, $P(X = r)$ increases as n increases.
Which of the statements given above is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

91. What is the value of λ for which the vectors $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + \hat{j} - \hat{k}$, $\lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar ?
(a) 1 (b) 2
(c) 3 (d) 4

92. What is the equation of the plane through z-axis and parallel to the line $\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$?
(a) $x \cot\theta + y = 0$
(b) $x \tan\theta - y = 0$
(c) $x + y \cot\theta = 0$
(d) $x - y \tan\theta = 0$

93. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b ?
(a) 1 (b) 3
(c) 6 (d) 0

94. What is the least value of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 2.5]$?
(a) -3 (b) 8
(c) -19 (d) -16.5

95. What is the derivative of $x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$?
(a) $\sqrt{a^2 - x^2}$ (b) $2\sqrt{a^2 - x^2}$
(c) $\sqrt{x^2 - a^2}$ (d) $2\sqrt{x^2 - a^2}$

96. If $(-5, 4)$ divides the line segment between the coordinate axes in the ratio 1 : 2, then what is its equation ?
(a) $8x + 5y + 20 = 0$
(b) $5x + 8y - 7 = 0$
(c) $8x - 5y + 60 = 0$
(d) $5x - 8y + 57 = 0$

97. What is the value of
$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \cos 45^\circ & \sin 45^\circ \end{vmatrix} \times \begin{vmatrix} \cos 45^\circ & \cos 15^\circ \\ \sin 45^\circ & \sin 15^\circ \end{vmatrix} ?$$
(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{2}$
(c) $-\frac{1}{4}$ (d) $-\frac{3}{4}$

98. Out of 32 persons, 30 invest in National Savings Certificates and 17 invest in shares. What is the number of persons who invest in both ?
(a) 13 (b) 15
(c) 17 (d) 19

99. What are the equations of the directrices of the ellipse $25x^2 + 16y^2 = 400$?
(a) $3x \pm 25 = 0$
(b) $3y \pm 25 = 0$
(c) $x \pm 15 = 0$
(d) $y \pm 25 = 0$

100. Let A be an $n \times n$ matrix. If $\det(\lambda A) = \lambda^s \det(A)$, what is the value of s ?
(a) 0 (b) 1
(c) -1 (d) n

101. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let $P = (1, 2)$ and $Q = (2, 1)$. Which one of the following is correct ?
(a) Q lies inside C but outside E
(b) Q lies outside both C and E
(c) P lies inside both C and E
(d) P lies inside C but outside E.

102. What is the geometric interpretation of the identity $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$?
1. If the diagonals of a given parallelogram are used as sides of a second parallelogram, then the area of the second parallelogram is twice that of the given parallelogram.
2. If the semi-diagonals of a given parallelogram are used as sides of a second parallelogram, then the area of the second parallelogram is half that of the given parallelogram.
Select the correct answer using the code given below
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

103. What is $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\, dx$?
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) 0

104. The function $f(x) = \frac{x}{x^2 + 1}$ from R to R is
(a) one-one as well as onto
(b) onto but not one-one

(c) neither one-one nor onto
(d) one-one but not onto

105. If A be a real skew-symmetric matrix of order n such that $A^2 + I = 0$, I being the identity matrix of the same order as that of A, then what is the order of A ?
(a) 3 (b) Odd
(c) Prime number (d) Even

106. The geometric mean of three numbers was computed as 6. It was subsequently found that, in this computation, a number 8 was wrongly read as 12. What is the correct geometric mean ?
(a) 4 (b) $\sqrt[3]{5}$
(c) $2\sqrt[3]{18}$ (d) None of these

107. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [a_{ij}]$, where $i, j = 1, 2$. If its inverse matrix is $[b_{ij}]$, what is b_{22} ?
(a) -2 (b) 1
(c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

108. The angle A lies in the third quadrant and it satisfies the equation $4(\sin^2 x + \cos x) = 1$. What is the measure of the angle A ?
(a) 225° (b) 240°
(c) 210° (d) None of these

109. What is the area enclosed between the curves $y^2 = 12x$ and the lines $x = 0$ and $y = 6$?
(a) 2 sq unit
(b) 4 sq unit
(c) 6 sq unit
(d) 8 sq unit

110. In a triangle ABC, $BC = \sqrt{39}$, $AC = 5$ and $AB = 7$. What is the measure of the angle A ?
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

111. What is the modulus of $\left|\frac{1+2i}{1-(1-i)^2}\right|$?
(a) 1 (b) $\sqrt{5}$
(c) $\sqrt{3}$ (d) 5

112. If the line through the points $A(k, 1, -1)$ and $B(2k, 0, 2)$ is perpendicular to the line through the points B and $C(2+2k, k, 1)$, then what is the value of k ?
(a) -1 (b) 1
(c) -3 (d) 3

113. What is $\int \frac{1}{1+e^x}\,dx$ equal to ?
(a) $x - \log x + c$
(b) $x - \log(\tan x) + c$
(c) $x - \log(1 + e^x) + c$
(d) $\log(1 + e^x) + c$
where c is a constant of integration.

114. The function $f(x) = x \operatorname{cosec} x$ is
(a) continuous for all values of x
(b) discontinuous everywhere
(c) continuous for all x except at $x = n\pi$, where n is an integer
(d) continuous for all x except at $x = n\pi/2$, where n is an integer

115. What is the solution of the differential equation $a\left(x\frac{dy}{dx} + 2y\right) = xy\frac{dy}{dx}$?
(a) $x^2 = kye^{\frac{y}{a}}$
(b) $yx^2 = kye^{\frac{y}{a}}$
(c) $y^2x^2 = kye^{\frac{y^2}{a}}$
(d) None of the above

116. A vector $\vec{b}$ is collinear with the vector $\vec{a} = (2, 1, -1)$ and satisfies the condition $\vec{a} \cdot \vec{b} = 3$. What is $\vec{b}$ equal to ?
(a) (1, 1/2, −1/2)
(b) (2/3, 1/3, −1/3)
(c) (1/2, 1/4, −1/4)
(d) (1, 1, 0)

117. What is the least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$?
(a) 16 (b) 12
(c) 8 (d) 4

118. The vectors $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{b} = \hat{k}$, $\vec{c}$ are such that they form a right handed system. What is $\vec{c}$ equal to ?
(a) $\hat{j}$ (b) $y\hat{j} - x\hat{k}$
(c) $y\hat{i} - x\hat{j}$ (d) $x\hat{i} - y\hat{j}$

119. If $x = t^2$, $y = t^3$, then what is $\frac{d^2y}{dx^2}$ equal to ?
(a) 1 (b) $\frac{3}{2t}$
(c) $\frac{3}{4t}$ (d) $\frac{3}{2}$

120. What is $\int_{-\pi/4}^{\pi/4} \tan^3 x\,dx$ equal to ?
(a) $\sqrt{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) 0

Answers : General Studies

1	(a)	51	(d)	101	(d)
2	(a)	52	(c)	102	(c)
3	(c)	53	(b)	103	(c)
4	(b)	54	(a)	104	(d)
5	(b)	55	(a)	105	(a)
6	(c)	56	(b)	106	(c)
7	(c)	57	(b)	107	(d)
8	(a)	58	(a)	108	(c)
9	(b)	59	(a)	109	(c)
10	(b)	60	(b)	110	(b)
11	(d)	61	(b)	111	(a)
12	(c)	62	(d)	112	(d)
13	(b)	63	(b)	113	(c)
14	(c)	64	(d)	114	(b)
15	(c)	65	(a)	115	(d)
16	(a)	66	(d)	116	(d)
17	(c)	67	(c)	117	(d)
18	(a)	68	(a)	118	(b)
19	(c)	69	(c)	119	(c)
20	(a)	70	(d)	120	(d)
21	(b)	71	(b)		
22	(c)	72	(d)		
23	(c)	73	(c)		
24	(d)	74	(c)		
25	(a)	75	(d)		
26	(a)	76	(b)		
27	(c)	77	(b)		
28	(c)	78	(c)		
29	(c)	79	(a)		
30	(d)	80	(b)		
31	(c)	81	(d)		
32	(b)	82	(a)		
33	(c)	83	(c)		
34	(c)	84	(d)		
35	(b)	85	(a)		
36	(b)	86	(b)		
37	(b)	87	(c)		
38	(c)	88	(b)		

39	(c)	89	(a)
40	(a)	90	(c)
41	(b)	91	(a)
42	(d)	92	(b)
43	(c)	93	(c)
44	(c)	94	(c)
45	(c)	95	(a)
46	(d)	96	(c)
47	(d)	97	(c)
48	(b)	98	(b)
49	(a)	99	(b)
50	(a)	100	(d)