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UPSC NDA Exam 2009-I Mathematics Solved
Paper

MATHEMATICS

- Let α, γ be the roots of $Ax^2 - 4x + 1 = 0$ and β, δ be the roots of $Bx^2 - 6x + 1 = 0$. If $\alpha, \beta, \gamma, \delta$ are in HP, then what are the values of A and B respectively?
 (a) 3, 8 (b) -3, -8
 (c) 3, -8 (d) -3, 8
- Under which one of the following condition does the system of equations

$$\begin{aligned} kx + y + z &= k - 1 \\ x + ky + z &= k - 1 \\ x + y + kz &= k - 1 \end{aligned}$$
 have no solution?
 (a) $k = 1$ (b) $k \neq -2$
 (c) $k = 1$ or $k = -2$ (d) $k = -2$
- If the sides of a triangle are 6 cm, 10 cm and 14 cm, then what is the largest angle included by the sides?
 (a) 90° (b) 120°
 (c) 135° (d) 150°
- If X and Y are any two non-empty sets, then what is $(X - Y)'$ equal to?
 (a) $X' - Y'$ (b) $X' \cap Y'$
 (c) $X' \cup Y'$ (d) $X - Y'$
- For finding the area of a triangle ABC , which of the following entities are required?
 (a) Angles A, B and side a
 (b) Angles A, B and side b
 (c) Angles A, B and side c
 (d) Either (a) or (b) or (c)
- Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where a, b are natural numbers, then which one of the following is correct?
 (a) There exist more than one but finite number of B 's such that $AB = BA$
 (b) There exists exactly one B such that $AB = BA$
 (c) There exist infinitely many B 's such that $AB = BA$
 (d) There cannot exist any B such that $AB = BA$
- Consider a matrix $M = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{bmatrix}$ and the following statements
 Statement A : Inverse of M exists.
 Statement B : $k \neq 0$
 Which one of the following in respect of the above matrix and statement is correct?
 (a) A implies B, but B does not imply A
 (b) B implies A, but A does not imply B
 (c) Neither A implies B nor B implies A
 (d) A implies B as well as B implies A
- If $2^x + 3^y = 17$ and $2^{x+2} - 3^{y+1} = 5$, then what is the value of x ?
 (a) 3 (b) 2
 (c) 1 (d) 0
- If $P(32, 6) = kC(32, 6)$, then what is the value of k ?
 (a) 6 (b) 32
 (c) 120 (d) 720
- What is $(\sqrt{3}+i)/(1+\sqrt{3}i)$ equal to?
 (a) $1+i$ (b) $1-i$
 (c) $\sqrt{3}(1-i)/2$ (d) $(\sqrt{3}-i)/2$
- What is the binary equivalent of decimal number $(0.8125)_{10}$?
 (a) $(0.1101)_2$ (b) $(0.1001)_2$
 (c) $(0.1111)_2$ (d) $(0.1011)_2$
- If $\begin{vmatrix} y & x & y+z \\ x & y & x+y \\ x & z & z+x \end{vmatrix} = 0$, then which one of the following is correct?
 (a) Either $x+y = z$ or $x = y$
 (b) Either $x+y = -z$ or $x = z$
 (c) Either $x+z = y$ or $z = y$
 (d) Either $z+y = x$ or $x = y$
- What is the value of k , if

$$\begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$
?
 (a) 1 (b) -1
 (c) 2 (d) 0
- What is the number of proper subsets of a given finite set with n elements?
 (a) $2n-1$ (b) $2n-2$
 (c) 2^n-1 (d) 2^n-2
- If $(x+a)$ is a factor of both the quadratic polynomials x^2+px+q and x^2+lx+m , where p, q, l and m are constants, then which one of the following is correct?
 (a) $a = (m-q)/(l-p)$ ($l \neq p$)
 (b) $a = (m+q)/(l+p)$ ($l \neq -p$)
 (c) $l = (m-q)/(a-p)$ ($a \neq p$)
 (d) $p = (m-q)/(a-l)$ ($a \neq l$)
- If A, B and C are three finite sets, then what is $[(A \cup B) \cap C]'$ equal to?
 (a) $A' \cup B' \cap C'$
 (b) $A' \cap B' \cap C'$
 (c) $A' \cap B' \cup C'$
 (d) $A \cap B \cap C$
- What is the value of $\tan(-1575^\circ)$?
 (a) 1 (b) $1/2$
 (c) 0 (d) -1
- For which acute angle θ , $\operatorname{cosec}^2 \theta = 3\sqrt{3} \cot \theta - 5$?
 (a) $\frac{5\pi}{12}$ (b) $\frac{\pi}{3}$
 (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$
- If $\tan^2 \theta = 2 \tan^2 \phi + 1$, then which one of the following is correct?
 (a) $\cos(2\theta) = \cos(2\phi) - 1$
 (b) $\cos(2\theta) = \cos(2\phi) + 1$
 (c) $\cos(2\theta) = [\cos(2\phi) - 1]/2$
 (d) $\cos(2\theta) = [\cos(2\phi) + 1]/2$

20. Which one of the following is correct in respect of the matrix
- $$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} ?$$
- (a) A^{-1} does not exist (b) $A = (-1)I$
 (c) A is a unit matrix (d) $A^2 = I$
21. The formula $\sin^{-1} \{2x(1-x^2)\} = 2\sin^{-1} x$ is true for all values of x lying in the interval
- (a) $[-1, 1]$ (b) $[0, 1]$
 (c) $[-1, 0]$ (d) $\{-1/\sqrt{2}, 1/\sqrt{2}\}$
22. What is the value of $1 - \sin 10^\circ \sin 50^\circ \sin 70^\circ$?
- (a) $1/8$ (b) $3/8$
 (c) $5/8$ (d) $7/8$
23. The sines of two angles of a triangle are equal to $5/13$ and $99/101$. What is the cosine of the third angle ?
- (a) $255/1313$ (b) $265/1313$
 (c) $275/1313$ (d) $770/1313$
24. After subtending an angle of 1000° from its initial position, the revolving line will be situated in which one of the following quadrants ?
- (a) First quadrant (b) Second quadrant
 (c) Third quadrant (d) Fourth quadrant
25. One radian is approximately equal to which one of the following ?
- (a) 90° (b) 180° (c) 57° (d) 47°
26. If $\cot(x+y) = 1/\sqrt{3}$, $\cot(x-y) = \sqrt{3}$, then what are the smallest positive values of x and y respectively ?
- (a) $45^\circ, 30^\circ$ (b) $30^\circ, 45^\circ$
 (c) $15^\circ, 60^\circ$ (d) $45^\circ, 15^\circ$
27. If $\sin A = 1/\sqrt{5}$, $\cos B = 3/\sqrt{10}$; A, B being positive acute angles, then what is $(A+B)$ equal to ?
- (a) $\pi/6$ (b) $\pi/4$
 (c) $\pi/3$ (d) $\pi/2$
28. If $\sin^{-1}\left(\frac{2x}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then what is the value of x ?
- (a) a/b (b) ab
 (c) b/a (d) $\frac{a-b}{1+ab}$
29. $x = \sin \theta \cos \theta$ and $y = \sin \theta + \cos \theta$ are satisfied by which one of the following equations ?
- (a) $y^2 - 2x = 1$ (b) $y^2 + 2x = 1$
 (c) $y^2 - 2x = -1$ (d) $y^2 + 2x = -1$
30. A man observes the elevation of a balloon to be 30° . He, then walks 1 km towards the balloon and finds that the elevation is 60° . What is the height of the balloon ?
- (a) $1/2$ km (b) $\sqrt{3}/2$ km
 (c) $1/3$ km (d) 1 km
31. The points $(1, 3, 4)$, $(-1, 6, 10)$, $(-7, 4, 7)$ and $(-5, 1, 1)$ are the vertices of a
- (a) rhombus (b) rectangle
 (c) parallelogram (d) square
32. What is the number of planes passing through three non-collinear points ?
- (a) 3 (b) 2 (c) 1 (d) 0
33. What is the angle between the lines $x+z=0$, $y=0$ and $20x=15y=12z$?
- (a) $\cos^{-1}(1/5)$ (b) $\cos^{-1}(1/7)$
 (c) $\sin^{-1}(1/5)$ (d) $\sin^{-1}(1/7)$
34. What is the eccentricity of an ellipse, if its latusrectum is equal to one-half of its minor axis ?
- (a) $1/4$ (b) $1/2$ (c) $\sqrt{3}/4$ (d) $\sqrt{3}/2$
35. Under what condition does the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represent a real sphere ?
- (a) $u^2 + v^2 + w^2 = d^2$ (b) $u^2 + v^2 + w^2 > d$
 (c) $u^2 + v^2 + w^2 < d$ (d) $u^2 + v^2 + w^2 < d^2$
36. Which one of the following vectors of magnitude $\sqrt{51}$ makes equal angles with three vectors
- $$\vec{a} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}, \vec{b} = \frac{-4\hat{i} - 3\hat{k}}{5} \text{ and } \vec{c} = \hat{j} ?$$
- (a) $5\hat{i} - \hat{j} - 5\hat{k}$ (b) $5\hat{i} + \hat{j} + 5\hat{k}$
 (c) $-5\hat{i} - \hat{j} + 5\hat{k}$ (d) $5\hat{i} + 5\hat{j} - \hat{k}$
37. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then what is the value of $\vec{a} \cdot \vec{b}$?
- (a) 4 (b) 6 (c) 8 (d) 10
38. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then which one of the following is correct ?
- (a) \vec{a} is parallel to \vec{b}
 (b) \vec{a} is perpendicular to \vec{b}
 (c) $\vec{a} = \vec{b}$
 (d) Both \vec{a} and \vec{b} are unit vectors
39. If $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then what is $(\vec{b} - \vec{a}) \cdot (3\vec{a} + \vec{b})$ equal to ?
- (a) 106 (b) -106
 (c) 53 (d) -53
40. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of points A, B, C respectively. Under which one of the following conditions are the points A, B, C collinear ?
- (a) $\vec{a} \times \vec{b} = \vec{0}$
 (b) $\vec{b} \times \vec{c}$ is parallel to $\vec{a} \times \vec{b}$
 (c) $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b} \times \vec{c}$
 (d) $(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \vec{0}$
41. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$, then what is $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$ equal to ?
- (a) $2\hat{i} + 3\hat{j} - \hat{k}$ (b) $2\hat{i} - 3\hat{j} - \hat{k}$
 (c) $3\hat{i} + \hat{j} + \hat{k}$ (d) $\vec{0}$
42. If A, B, C are any three arbitrary events, then which one of the following expressions shows that both A and B occur but not C ?
- (a) $A \cap \bar{B} \cap \bar{C}$ (b) $A \cap B \cap \bar{C}$
 (c) $A \cap B \cap C$ (d) $A \cap \bar{B} \cap C$

43. The average sales and standard deviation of sales for four months for a company are as follows

	Month 1	Month 2	Month 3	Month 4
Average sales	30	57	82	28
Standard deviation of sales	2	3	4	2

- During which month are the sales most consistent ?
 (a) Month 1 (b) Month 2
 (c) Month 3 (d) Month 4
44. By Baye's theorem, which one of the following probabilities is calculated ?
 (a) Prior probability (b) Likelihood probability
 (c) Posterior probability (d) Conditional probability
45. Given that $P(A) = 1/3$, $P(B) = 1/4$, $P(A/B) = 1/6$, then what is $P(B/A)$ equal to ?
 (a) $1/4$ (b) $1/8$ (c) $3/4$ (d) $1/2$
46. If A and B are two mutually exclusive and exhaustive events with $P(B) = 3P(A)$, then what is the value of $P(B)$?
 (a) $3/4$ (b) $1/4$ (c) $1/3$ (d) $2/3$
47. Two dice are thrown. What is the probability that the sum of the faces equals or exceeds 10 ?
 (a) $1/12$ (b) $1/4$ (c) $1/3$ (d) $1/6$
48. For a binomial distribution $B(n, p)$, $np = 4$ and variance $npq = 4/3$. What is the probability $P(x \geq 5)$ equal to ?
 (a) $(2/3)^6$ (b) $2^5/3^6$ (c) $(1/3)^6$ (d) $2^8/3^6$
49. The harmonic mean of two numbers is 21.6. If one of the numbers is 27, then what is the other number ?
 (a) 16.2 (b) 17.3 (c) 18 (d) 20
50. The marks scored by two students A and B in six subjects are given below
- | | | | | | | |
|-----|----|----|----|----|----|----|
| A | 71 | 56 | 55 | 75 | 54 | 49 |
| B | 55 | 74 | 83 | 54 | 38 | 52 |
- Which one of the following statements is most appropriate ?
 (a) The average scores of A and B are same but A is consistent
 (b) The average scores of A and B are not same but A is consistent
 (c) The average scores of A and B are same but B is consistent
 (d) The average scores of A and B are not same but B is consistent
51. In a factory, there are 30 men and 20 women employees. If the average salary of men is Rs 4050 and the average salary of all the employees is Rs 3550, then what is the average salary of women ?
 (a) Rs 3800 (b) Rs 3300
 (c) Rs 3000 (d) Rs 2800
52. What is the standard deviation of numbers 7, 9, 11, 13, 15 ?
 (a) 2.2 (b) 2.4
 (c) 2.6 (d) 2.8
53. When a card is drawn from a well shuffled pack of cards, what is the probability of getting a Queen ?
 (a) $2/13$ (b) $1/13$
 (c) $1/26$ (d) $1/52$
54. If the monthly expenditure pattern of a person who earns a monthly salary of Rs 15000 is represented in a pie diagram, then the sector angle of an item on transport expenses

measures 15° . What is his monthly expenditure on transport ?

- (a) Rs 450
 (b) Rs 625
 (c) Rs 675
 (d) Cannot be computed from the given data
55. If $\sum_{i=1}^n (x_i - 2) = 110$, $\sum_{i=1}^n (x_i - 5) = 20$, then what is the mean ?
 (a) $11/2$ (b) $2/11$ (c) $17/3$ (d) $17/9$
56. Let $f: R \rightarrow R$ be a function defined as $f(x) = x|x|$; for each $x \in R$, R being the set of real numbers. Which one of the following is correct ?
 (a) f is one-one but not onto
 (b) f is onto but not one-one
 (c) f is both one-one and onto
 (d) f is neither one-one nor onto
57. What is the set of all points, where the function $f(x) = \frac{x}{1+|x|}$ is differentiable ?
 (a) $(-\infty, \infty)$ only (b) $(0, \infty)$ only
 (c) $(-\infty, 0) \cup (0, \infty)$ only (d) $(-\infty, 0)$ only
58. Let $y(x) = ax^n$ and δy denote small change in y . What is limit of $\frac{\delta y}{\delta x}$ as $\delta x \rightarrow 0$?
 (a) 0 (b) 1
 (c) anx^{n-1} (d) $ax^n \log(ax)$
59. What is the solution of the differential equation $\frac{dy}{dx} = e^{x-y} (e^y - x - e^y)$?
 (a) $y = x - e^x + c$ (b) $y = x + e^x + c$
 (c) $y = e^{x-y} - e^y + c$ (d) None of these
60. What is the area of the triangle formed by the lines $y - x = 0$, $y + x = 0$, $x = c$?
 (a) $c/2$ (b) c^2 (c) $2c^2$ (d) $c^2/2$
61. If A and B are subsets of a set X , then what is $\{A \cap (X - B)\} \cup B$ equal to ?
 (a) $A \cup B$ (b) $A \cap B$
 (c) A (d) B
62. The total number of subsets of a finite set A has 56 more elements, then the total number of subsets of another finite set B . What is the number of elements in the set A ?
 (a) 5 (b) 6 (c) 7 (d) 8
63. What is the smallest natural number n such that $n!$ is divisible by 990 ?
 (a) 9 (b) 11 (c) 33 (d) 99
64. Which one of the following is correct ?
 (a) $A \times (B - C) = (A - B) \times (A - C)$
 (b) $A \times (B - C) = (A \times B) - (A \times C)$
 (c) $A \cap (B \cup C) = (A \cap B) \cup C$
 (d) $A \cup (B \cap C) = (A \cup B) \cap C$
65. In an examination out of 100 students, 75 passed in English, 60 passed in Mathematics and 45 passed in both English and Mathematics. What is the number of students passed in exactly one of the two subjects ?
 (a) 45 (b) 60
 (c) 75 (d) 90

66. Let $R = \{x \mid x \in N, x \text{ is a multiple of } 3 \text{ and } x \leq 100\}$
 $S = \{x \mid x \in N, x \text{ is a multiple of } 5 \text{ and } x \leq 100\}$
 What is the number of elements in $(R \times S) \cap (S \times R)$?
 (a) 36 (b) 33 (c) 20 (d) 6
67. If $A = \{a, b, c\}$ and $R = \{(a, a), (a, b), (b, c), (b, b), (c, c), (c, a)\}$ is a binary relation on A , then which one of the following is correct?
 (a) R is reflexive and symmetric, but not transitive
 (b) R is reflexive and transitive, but not symmetric
 (c) R is reflexive, but neither symmetric nor transitive
 (d) R is reflexive, symmetric and transitive
68. If $\log_{10}(x+1) + \log_{10} 5 = 3$, then what is the value of x ?
 (a) 199 (b) 200 (c) 299 (d) 300
69. What is the value of $2 \log_8 2 - \frac{\log_3 9}{3}$?
 (a) 0 (b) 1 (c) $8/3$ (d) $16/3$
70. Which one of the following is one of the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$?
 (a) $(c-a)/(b-c)$ (b) $(a-b)/(b-c)$
 (c) $(b-c)/(a-b)$ (d) $(c-a)/(a-b)$
71. What is the value of x satisfying the equation $16 \left(\frac{a-x}{a+x} \right)^3 = \frac{a+x}{a-x}$?
 (a) $a/2$ (b) $a/3$ (c) $a/4$ (d) 0
72. If α, β are the roots of the equation $2x^2 - 2(1+n^2)x + (1+n^2+n^4) = 0$, then what is the value of $\alpha^2 + \beta^2$?
 (a) $2n^2$ (b) $2n^4$ (c) 2 (d) n^2
73. The roots of $Ax^2 + Bx + C = 0$ are r and s . For the roots of $x^2 + px + q = 0$ to be r^2 and s^2 , what must be the value of p ?
 (a) $(B^2 - 4AC)/A^2$ (b) $(B^2 - 2AC)/A^2$
 (c) $(2AC - B^2)/A^2$ (d) $B^2 - 2C$
74. What is the value of r , if $P(5, r) = P(6, r-1)$?
 (a) 9 (b) 5 (c) 4 (d) 2
75. What is the number of words formed from the letters of the word 'JOKE' so that the vowels and consonants alternate?
 (a) 4 (b) 8
 (c) 12 (d) None of these
76. If the sum of the first two terms and the sum of the first four terms of a geometric progression with positive common ratio are 8 and 80 respectively, then what is the 6th term?
 (a) 88 (b) 243 (c) 486 (d) 1458
77. What is the decimal equivalent of $(101.101)_2$?
 (a) $(5.225)_{10}$ (b) $(5.525)_{10}$
 (c) $(5.625)_{10}$ (d) $(5.65)_{10}$
78. If $x > 1$ and $\log_2 x, \log_3 x, \log_x 16$ are in GP, then what is x equal to?
 (a) 9 (b) 8 (c) 4 (d) 2
79. What is the term independent of x in the expansion of $(1+x+2x^3) \left(\frac{3x^{-2}}{2} - \frac{1}{3x} \right)^9$?
 (a) $1/3$
 (b) $19/54$
 (c) $1/4$
 (d) No such term exists in the expansion

80. In a geometric progression with first term a and common ratio r , what is the arithmetic mean of first five terms?
 (a) $a, 2r$ (b) ar^2
 (c) $a(r^5 - 1)/(r - 1)$ (d) $a(r^5 - 1)/[5(r - 1)]$
81. If $2x = 3 + 5i$, then what is the value of $2x^3 + 2x^2 - 7x + 72$?
 (a) 4 (b) -4
 (c) 8 (d) -8
82. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, then what is $A(\text{adj } A)$ equal to?
 (a) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$
 (c) $\begin{bmatrix} 1 & 10 \\ 10 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix}$
83. What is the inverse of $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$?
 (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
84. Consider the following statements in respect of symmetric matrices A and B
 1. AB is symmetric.
 2. $A^2 + B^2$ is symmetric.
 Which of the above statement(s) is/are correct?
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
- Directions (Q. 85-90) :** The following six items consists of two statements, one labelled the Assertion (A) and the other labelled the Reason (R). You are to examine these two statements carefully and decide if the Assertion (A) and Reason (R) are individually true and if so, whether the reason is a correct explanation of the Assertion. Select your answer to these items using the codes given below.
- Codes :**
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true
85. **Assertion (A) :** The tangent to the curve $y = x^3 - x^2 - x + 2$ and $(1, 1)$ is parallel to the x -axis.
Reason (R) : The slope of the tangent to the curve at $(1, 1)$ is zero.
86. **Assertion (A) :** The work done when the force and displacement are perpendicular to each other is zero.
Reason (R) : The dot product $\vec{A} \cdot \vec{B}$ vanishes, is the vector \vec{A} and \vec{B} are perpendicular.

87. Assertion (A) : The probability of drawing either an ace or a king from a deck of card in a single draw is $\frac{2}{13}$.

Reason (R) : For two events E_1 and E_2 , which are not mutually exclusive probability is given by

$$P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

88. Assertion (A) : $\left(\frac{-1 + \sqrt{-3}}{2}\right)^{29} + \left(\frac{-1 - \sqrt{-3}}{2}\right)^{29} = -1$

Reason (R) : $\omega^2 = -1$

89. Assertion (A) : $M = \begin{bmatrix} 5 & 10 \\ 4 & 8 \end{bmatrix}$ is invertible.

Reason (R) : M is singular.

90. Assertion (A) : $\int \frac{e^x}{x} (1 + x \log x) dx + c = e^x \log x$

Reason (R) : $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

91. If $\sin^4 x - \cos^4 x = p$, then which one of the following is correct ?

- (a) $p = 1$ (b) $p = 0$
(c) $|p| > 1$ (d) $|p| \leq 1$

92. If $\cos \theta < \sin \theta$ and θ lies in the first quadrant, then which one of the following is correct ?

- (a) $0 < \theta < \pi/4$ (b) $\pi/4 < \theta < \pi/2$
(c) $0 < \theta < \pi/3$ (d) $\pi/3 < \theta < \pi/2$

93. The angle of elevation from a point on the bank of a river of the top of a temple on the other bank is 45° . Retreating 50 m, the observer finds the new angle of elevation as 30° . What is the width of the river ?

- (a) 50 m (b) $50\sqrt{3}$ m
(c) $50(\sqrt{3} - 1)$ m (d) 100 m

94. If $\sin^2 x + \sin^2 y = 1$, then what is the value of $\cot(x + y)$?

- (a) 1 (b) $\sqrt{3}$ (c) 0 (d) $1/\sqrt{3}$

95. What is the value of $\cos 10^\circ + \cos 110^\circ + \cos 130^\circ$?

- (a) -1 (b) 0 (c) 1 (d) 2

96. If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then what is $y(x)$ equal to ?

- (a) $e^{\frac{(1+x)^2}{2}} - 1$ (b) $e^{\frac{(1-x)^2}{2}}$
(c) $\log(1+x) - 1$ (d) $\log(1-x)$

97. What is the value of $\int_0^{\pi/2} \log(\tan x) dx$?

- (a) 0 (b) 1 (c) -1 (d) $\pi/4$

98. What is $\int \tan^2 x \sec^4 x dx$ equal to ?

- (a) $\frac{\sec^5 x}{5} + \frac{\sec^3 x}{3} + c$ (b) $\frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c$
(c) $\frac{\tan^5 x}{5} + \frac{\sec^3 x}{3} + c$ (d) $\frac{\sec^5 x}{5} + \frac{\tan^3 x}{3} + c$

99. What is $\lim_{x \rightarrow 0} \frac{\sin^2 ax}{bx}$ (a, b are constants) equal to ?

- (a) 0 (b) $a, \frac{a^2}{b}$
(c) a^2/b (d) Does not exist

100. If $f(x) = \tan x + e^{-2x} - 7x^3$, then what is the value of $f'(0)$?

- (a) -2 (b) -1 (c) 0 (d) 3

101. What are the degree and order respectively of differential equation of the family of rectangular hyperbolas whose axes of symmetry are the coordinate axes ?

- (a) 1, 1 (b) 1, 2 (c) 2, 1 (d) 2, 2

102. The function $f(x) = x^2 - 2x$ increases for all

- (a) $x > -1$ only (b) $x < -1$ only
(c) $x > 1$ only (d) $x < 1$ only

103. What is $\int_0^1 x(1-x)^n dx$ equal to ?

- (a) $\frac{1}{n(n+1)}$ (b) $\frac{1}{(n+1)(n+2)}$
(c) 1 (d) 0

104. Let a and b be two distinct roots of a polynomial equation $f(x) = 0$. Then there exists at least one root lying between a and b of the polynomial equation

- (a) $f(x) = 0$ (b) $f'(x) = 0$
(c) $f''(x) = 0$ (d) None of these

105. If $3^x + 3^y = 3^{x+y}$, then what is $\frac{dy}{dx}$ equal to ?

- (a) $\frac{3^{x+y} - 3^x}{3^y}$ (b) $\frac{3^{x-y}(3^y - 1)}{1 - 3^x}$
(c) $\frac{3^x + 3^y}{3^x - 3^y}$ (d) $\frac{3^x + 3^y}{1 + 3^{x+y}}$

106. What is $\int \sec x^\circ dx$ equal to ?

- (a) $\log(\sec x^\circ + \tan x^\circ) + c$
(b) $\frac{\pi \log \tan\left(\frac{\pi}{4} + \frac{\pi}{2}\right)}{180^\circ} + c$
(c) $\frac{180^\circ \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)}{\pi} + c$
(d) $\frac{180^\circ \log \tan\left(\frac{\pi}{4} + \frac{\pi x}{360^\circ}\right)}{\pi} + c$

107. The profit function, in rupees, of a firm selling x items ($x \geq 0$) per week is given by $P(x) = -3500 + (400 - x)x$. How many items should the firm sell so that the firm has maximum profit ?

- (a) 400 (b) 300 (c) 200 (d) 100

108. A stone thrown vertically upward satisfies the equation $s = 64t - 16t^2$, where s is in meter and t is in second. What is the time required to reach the maximum height ?

- (a) 1 s (b) 2 s (c) 3 s (d) 4 s

109. If $f(x) = 3x^2 + 6x - 9$, then

- (a) $f(x)$ is increasing in $(-1, 3)$
(b) $f(x)$ is decreasing in $(3, \infty)$
(c) $f(x)$ is increasing in $(-\infty, -1)$
(d) $f(x)$ is decreasing in $(-\infty, -1)$

110. If $f(x) = \sin^2 x^2$, then what is $f'(x)$ equal to ?

- (a) $4x \sin(x^2) \cos(x^2)$ (b) $2 \sin(x^2) \cos(x^2)$
(c) $4 \sin(x^2) \sin^2 x$ (d) $2x \cos^2(x^2)$

111. If $f(x) = \cos x$, $g(x) = \log x$ and $y = (g \circ f)(x)$, then what is the value of $\frac{dy}{dx}$ at $x = 0$?

- (a) 0 (b) 1 (c) -1 (d) 2

112. If $f(x) = \begin{cases} 3x - 4, & 0 \leq x \leq 2 \\ 2x + \lambda, & 2 < x \leq 3 \end{cases}$

is continuous at $x = 2$, then what is the value of λ ?

- (a) 1 (b) -1 (c) 2 (d) -2

113. If $x \cos \theta + y \sin \theta = 2$ is perpendicular to the line $x - y = 3$, then what is one of the value of θ ?

- (a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi/3$

114. If x -axis is tangent to the circle $x^2 + y^2 + 2gx + 2fy + k = 0$, then which one of the following is correct?

(a) $g^2 = k$ (b) $g^2 = f$

(c) $f^2 = k$ (d) $f^2 = g$

115. What is the sum of focal radii of any point on an ellipse equal to?

- (a) Length of latusrectum
 (b) Length of major-axis
 (c) Length of minor-axis
 (d) Length of semi-latusrectum

116. What does an equation of the first degree containing one arbitrary parameter passing through a fixed point represent?

- (a) Circle (b) Straight line
 (c) Parabola (d) Ellipse

117. What is the foot of the perpendicular from the point $(2, 3)$ on the line $x + y - 11 = 0$?

- (a) $(1, 10)$ (b) $(5, 6)$
 (c) $(6, 5)$ (d) $(7, 4)$

118. Consider the following statements

1. The equation to a straight line parallel to the axis of x is $y = d$, where d is a constant.

2. The equation to the axis of x is $x = 0$.

Which of the statement(s) given above is/are correct?

- (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

119. What is the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$?

- (a) $\pi/2$ (b) $\pi/3$
 (c) $\pi/4$ (d) $\pi/6$

120. What is the equation of a plane through the x -axis and passing through the point $(1, 2, 3)$?

- (a) $x + y + z = 6$ (b) $x = 1$
 (c) $y + z = 5$ (d) $z + y = 1$

Answers : Mathematics

1	(d)	51	(d)	101	(a)
2	(c)	52	(d)	102	(c)
3	(b)	53	(b)	103	(b)
4	(c)	54	(b)	104	(b)
5	(c)	55	(c)	105	(b)
6	(c)	56	(c)	106	(d)
7	(d)	57	(a)	107	(c)
8	(a)	58	(a)	108	(b)
9	(d)	59	(a)	109	(d)
10	(d)	60	(b)	110	(a)
11	(a)	61	(a)	111	(a)
12	(b)	62	(b)	112	(d)
13	(a)	63	(b)	113	(b)
14	(c)	64	(b)	114	(a)
15	(a)	65	(a)	115	(b)
16	(c)	66	(a)	116	(b)
17	(a)	67	(c)	117	(b)
18	(c)	68	(a)	118	(a)
19	(c)	69	(a)	119	(b)
20	(d)	70	(b)	120	(b)
21	(d)	71	(b)		
22	(d)	72	(d)		
23	(a)	73	(c)		
24	(d)	74	(c)		
25	(c)	75	(b)		
26	(d)	76	(c)		
27	(d)	77	(c)		
28	(d)	78	(a)		
29	(a)	79	(d)		
30	(b)	80	(d)		
31	(a)	81	(a)		
32	(c)	82	(b)		
33	(c)	83	(b)		
34	(d)	84	(b)		
35	(b)	85	(a)		
36	(a)	86	(b)		
37	(b)	87	(b)		
38	(b)	88	(c)		

39	(b)	89	(d)
40	(d)	90	(a)
41	(d)	91	(d)
42	(b)	92	(b)
43	(c)	93	(c)
44	(d)	94	(c)
45	(b)	95	(b)
46	(b)	96	(a)
47	(d)	97	(a)
48	(d)	98	(b)
49	(c)	99	(a)
50	(b)	100	(b)