JAGRAN Josh
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UPSC NDA Exam 2008-II Mathematics Solved Paper

Mathematics

1. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(b > a)$. Then, which one of the following is correct ?
   (a) Real foci do not exist
   (b) Foci are $(\pm ae, 0)$
   (c) Foci are $(\pm be, 0)$
   (d) Foci are $(0, \pm be)$

2. Consider the parabolas $S_1 = y^2 - 4ax = 0$ and $S_2 = y^2 - 4bx = 0$. S_2 will contain S_1, if
   (a) $a > b > 0$
   (b) $b > a > 0$
   (c) $a > 0, b < 0$ but $|b| > a$
   (d) $a < 0, b > 0$ but $b > |a|$

3. What are the coordinates of the point equidistant from the four points $(0, 0, 0), (2, 0, 0), (0, 4, 0), (0, 0, 6)$?
   (a) $(1, 2, 3)$
   (b) $(2, 3, 1)$
   (c) $(3, 1, 2)$
   (d) $(1, 3, 2)$

4. The angle between the lines with direction ratios $(1, 0, \pm \cos \alpha)$ is $60°$. What is the value of α ?
   (a) $\cos^{-1}(1/\sqrt{2})$
   (b) $\cos^{-1}(1/\sqrt{3})$
   (c) $\cos^{-1}(1/3)$
   (d) $\cos^{-1}(1/2)$

5. The line passing through $(1, 2, 3)$ and having direction ratios given by $< 1, 2, 3 >$ cuts the x-axis at a distance k from origin. What is the value of k ?
   (a) 0
   (b) 1
   (c) 2
   (d) 3

6. The equation $by + cz + d = 0$ represents a plane parallel to which one of the following ?
   (a) x-axis
   (b) y-axis
   (c) z-axis
   (d) None of these

7. Which one of the following planes is normal to the plane $3x + y + z = 5$?
   (a) $x + 2y + z = 6$
   (b) $x - 2y + z = 6$
   (c) $x + 2y - z = 6$
   (d) $x - 2y - z = 6$

8. If the radius of the sphere $x^2 + y^2 + z^2 - 6x - 8y + 10z + \lambda = 0$ is unity, what is the value of λ ?
   (a) 49
   (b) 7
   (c) -49
   (d) -7

9. Curve of intersection of two spheres is
   (a) an ellipse
   (b) a circle
   (c) a parabola
   (d) None of these

10. The equation of the circle which touches the axes at a distance 5 from the origin is $y^2 + x^2 - 2\alpha x - 2\alpha y + \alpha^2 = 0$. What is the value of α ?
    (a) 4
    (b) 5
    (c) 6
    (d) 7

11. If the number of terms of an AP is $(2n + 1)$, then what is the ratio of the sum of the odd terms to the sum of even terms ?
    (a) $\frac{n}{n+1}$
    (b) $\frac{n^2}{n+1}$
    (c) $\frac{n+1}{n}$
    (d) $\frac{n+1}{2n}$

12. If $\sin x + \sin y = a$, $\cos x + \cos y = b$, then what is the value of $\cos(x - y)$?
    (a) $a^2 - 1$
    (b) $b^2 - 1$
    (c) $\frac{1}{2}(a^2 + b^2 - 2)$
    (d) $\frac{1}{2}(a^2 + b^2)$

13. What is $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 4A}}}$ equal to ?
    (a) $\cos A$
    (b) $\cos(2A)$
    (c) $2\cos(A/2)$
    (d) $\sqrt{2\cos A}$

14. If x is an integer and satisfies $9 < 4x - 1 \le 19$, then x is an element of which one of the following sets ?
    (a) $\{3, 4\}$
    (b) $\{2, 3, 4\}$
    (c) $\{3, 4, 5\}$
    (d) $\{2, 3, 4, 5\}$

15. A function f is defined by $f(x) = x + \frac{1}{x}$. Consider the following
    1. $(f(x))^2 = f(x^2) + 2$
    2. $(f(x))^3 = f(x^3) + 3f(x)$
    Which of the above is /are correct ?
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

16. Let $\vec{a} = (1, -2, 3)$ and $\vec{b} = (3, 1, 2)$ be two vectors and $\vec{c}$ be a vector of length l and parallel to $(\vec{a} + \vec{b})$. What is $\vec{c}$ equal to ?
    (a) $\frac{1}{\sqrt{14}}(-2, -3, 1)$
    (b) $\frac{1}{\sqrt{2}}(1, 0, 1)$
    (c) $\frac{1}{\sqrt{42}}(-5, -4, -1)$
    (d) None of these

17. If $\vec{r_1} = \lambda \hat{i} + 2\hat{j} + \hat{k}$, $\vec{r_2} = \hat{i} + (2 - \lambda)\hat{j} + 2\hat{k}$ are such that $|\vec{r_1}| > |\vec{r_2}|$, then λ satisfies which one of the following ?
    (a) $\lambda = 0$ only
    (b) $\lambda = 1$
    (c) $\lambda < 1$
    (d) $\lambda > 1$

18. If P, Q, R are the mid points of the sides AB, BC, CA, respectively of a triangle ABC and if $\vec{a}, \vec{p}, \vec{q}$ are the position vector of A, P, Q respectively, then what is the position vector of R ?
    (a) $2\vec{a} - (\vec{p} - \vec{q})$
    (b) $(\vec{p} - \vec{q}) - 2\vec{a}$
    (c) $\vec{a} - (\vec{p} - \vec{q})$
    (d) $\vec{a}/2 - (\vec{p} - \vec{q})/2$

19. If $a = x + \sqrt{x^2 + 1}$, then what is x equal to ?
    (a) $(1/2)(a + a^{-1})$
    (b) $(1/2)(a - a^{-1})$
    (c) $a + a^{-1}$
    (d) $a - a^{-1}$

20. A quadratic polynomial with two distinct roots has one real root. Then, the other root is
    (a) not necessarily real, if the coefficients are real
    (b) always imaginary
    (c) always real
    (d) real, if the coefficients are real.

21. If $f(x) = \log|x|$, $x \ne 0$, then what is $f'(x)$ equal to ?

(a) $\dfrac{1}{|x|}$ (b) $\dfrac{1}{x}$

(c) $\dfrac{-1}{x}$ (d) None of these

22. What is the equation of the curve whose slope at any point is equal to $2x$ and which passes through the origin ?

(a) $y(1-x) = x^2$ (b) $y^2(1+x^2) = x^4$

(c) $y^2 = (x+1)^2$ (d) $y = x^2$

23. What is the solution of the differential equation $-\operatorname{cosec}^2(x+y)\, dy = dx$?

(a) $y - c = \sin(x+y)$

(b) $x - c = \sin(x+y)$

(c) $y - c = \tan(x+y)$

(d) None of the above

24. What is the area of the ellipse $4x^2 + 9y^2 = 1$?

(a) 6π (b) $\dfrac{\pi}{36}$

(c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{\sqrt{6}}$

25. What is the value of $\sin(1110°)$?

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{\sqrt{3}}{2}$

26. If $y = \sin(m\sin^{-1}x)$, what is the value of d^2y/dx^2 at $x = 0$?

(a) m (b) m^2

(c) $m^2 + 2$ (d) None of these

27. $\lim\limits_{x \to 0} e^{-1/x}$ is equal to

(a) 0 (b) ∞

(c) e (d) dose not exist

28. What are the order and degree respectively of the differential equation
$\{(d^4y/dx^4)^3\}^{2/3} - 7x(d^3y/dx^3)^2 = 8$?

(a) $3, 2$ (b) $4, 3$

(c) $4, 2$ (d) $3, 3$

29. What is the solution of the differential equation $x\,dy - y\,dx = xy^2\,dx$?

(a) $yx^2 + 2x = 2cy$ (b) $y^2x + 2y = 2cx$

(c) $y^2x^2 + 2x = 2cy$ (d) None of these

30. The value of $\int_{-2}^{2}(ax^3 + bx + c)\,dx$ depends on which of the following ?

(a) Values of x only

(b) Values of each of a, b and c

(c) Value of c only

(d) Value of b only

31. Let $g : R \to R$ be a function such that, $g(x) = 2x + 5$. Then, what is $g^{-1}(x)$ equal to ?

(a) $\dfrac{x-5}{2}$ (b) $2x - 5$

(c) $x - \dfrac{5}{2}$ (d) $\dfrac{x}{2} + \dfrac{5}{2}$

32. Consider the following statements

1. $\lim\limits_{x \to 0}\dfrac{x^2}{x}$ exists.

2. $\left(\dfrac{x^2}{x}\right)$ is not continuous at $x = 0$.

3. $\lim\limits_{x \to 0}\dfrac{|x|}{x}$ does not exist.

Which of the statements given above are correct ?

(a) 1, 2 and 3   (b) 1 and 2 only

(c) 2 and 3 only   (d) 1 and 3 only

33. Let $f(x) = \dfrac{1}{1 - |1 - x|}$. Then, what is $\lim\limits_{x \to 0} f(x)$ equal to

(a) 0 (b) ∞

(c) 1 (d) -1

34. What is the value of $\lim\limits_{x \to \alpha} \dfrac{\sqrt{\alpha + 2x}}{\sqrt{3\alpha + x}} \cdot \dfrac{-\sqrt{3x}}{-2\sqrt{x}}$?

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{1}{(3\sqrt{3})}$

(c) $\dfrac{2}{(3\sqrt{3})}$ (d) $\dfrac{1}{\sqrt{3}}$

35. If $x^y = e^{x-y}$, then dy/dx is equal to which one of the following ?

(a) $\dfrac{(x-y)}{(1 + \log x)^2}$ (b) $\dfrac{y}{(1 + \log x)}$

(c) $\dfrac{(x+y)}{(1 + \log x)}$ (d) $\dfrac{(\log x)}{(1 + \log x)^2}$

36. What is the maximum value of the function $\log x - x$?

(a) -1 (b) 0

(c) 1 (d) ∞

37. A rectangular box with a cover is to have a square base. The volume is to be 10 cubic cm. The surface area of the box in terms of the side x is given by which one of the following functions ?

(a) $f(x) = (40/x) + 2x^2$ (b) $f(x) = (40/x) + x^2$

(c) $f(x) = (40/x) + x$ (d) $f(x) = (60/x) + 2x$

38. $f(x) = \cos x$ is monotonic decreasing under which one of the following conditions ?

(a) $0 < x < \dfrac{\pi}{2}$ only   (b) $\dfrac{\pi}{2} < x < \pi$ only

(c) $0 < x < \pi$ (d) $0 < x < 2\pi$

39. What is $\int (e^x + 1)^{-1}dx$ equal to ?

(a) $\ln(e^x + 1) + c$ (b) $\ln(e^{-x} + 1) + c$

(c) $-\ln(e^{-x} + 1) + c$ (d) $-(e^x + 1) + c$

40. What is $\int \dfrac{d\theta}{\sin^2\theta + 2\cos^2\theta - 1}$ equal to ?

(a) $\tan\theta + c$ (b) $\cot\theta + c$

(c) $\dfrac{1}{2}\tan\theta + c$ (d) $\dfrac{1}{2}\cot\theta + c$

41. What are the values of p which satisfy the equation $\int_{0}^{p}(3x^2 + 4x - 5)\,dx = p^3 - 2$?

(a) 1/2 and 2   (b) $-1/2$ and 2

(c) 1/2 and -2 (d) $-1/2$ and -2

42. What is $\int \sin x \log(\tan x)\,dx$ equal to ?

    (a) $\cos x \log \tan x + \log \tan(x/2) + c$
    (b) $-\cos x \log \tan x + \log \tan(x/2) + c$
    (c) $\cos x \log \tan x + \log \cot(x/2) + c$
    (d) $-\cos x \log \tan x + \log \cot(x/2) + c$

43. What does the solution of the differential equation $x\,dy - y\,dx = 0$ represent ?
    (a) Rectangular hyperbola
    (b) Straight line passing through origin
    (c) Parabola whose vertex is at origin
    (d) Circle whose centre is at origin

44. What is the order of the differential equation $\dfrac{dy}{dx} + y = \dfrac{1}{\left(\dfrac{dy}{dx}\right)}$?

    (a) -1 (b) 0
    (c) 1   (d) 2

45. Rate of growth of bacteria is proportional to the number of bacteria present at that time. If x is the number of bacteria present at any instant t, then which one of the following is correct ?
    (Take proportional constant equal to 1)
    (a) $x = \log t$ (b) $x = c\,e^t$
    (c) $e^x = t$ (d) $x = \sqrt{t}$

46. Equation of the hyperbola with eccentricity 3/2 and foci at $(\pm 2, 0)$ is $5x^2 - 4y^2 = k^2$. What is the value of k ?
    (a) 4/3   (b) 3/4
    (c) $(4/3)\sqrt{5}$ (d) $(3/4)\sqrt{5}$

47. What does the equation $x^3 y + xy^3 - xy = 0$ represent ?
    (a) A pair of straight lines only
    (b) A pair of straight lines and a circle
    (c) A rectangular hyperbola only
    (d) A rectangular hyperbola and a circle

48. The point of intersection of the two lines $2x + 3y + 4 = 0$ and $4x + 3y + 2 = 0$ is at a distance d from origin. What is the value of d ?
    (a) $\sqrt{2}$ (b) $\sqrt{3}$
    (c) $\sqrt{5}$ (d) $\sqrt{7}$

49. The line through the points (4, 3) and (2, 5) cuts off intercepts of lengths λ and μ on the axes. Which one of the following is correct ?
    (a) $\lambda > \mu$ (b) $\lambda < \mu$
    (c) $\lambda > -\mu$ (d) $\lambda = \mu$

50. What is the locus of a point which is equidistant from the points $(a+b,\, a-b)$ and $(b-a,\, a+b)$?
    (a) $bx - ay = 0$ (b) $bx + ay = 0$
    (c) $-ax + by = 0$ (d) $ax + by = 0$

51. If the sum of 'n' terms of an arithmetic progression is $n^2 - 2n$, then what is the nth term ?
    (a) $3n - n^2$ (b) $2n - 3$
    (c) $2n + 3$ (d) $2n - 5$

52. If $\sin \alpha$ and $\cos \alpha$ are the roots of the equation $px^2 + qx + r = 0$, then which one of the following is correct ?
    (a) $p^2 + q^2 - 2pr = 0$ (b) $p^2 - q^2 + 2pr = 0$
    (c) $(p+r)^2 = 2(p^2 + r^2)$ (d) $(p-r)^2 = q^2 + r^2$

53. If $a,\ 2a+2,\ 3a+3$ are in GP, then what is the fourth term of the GP ?
    (a) -13.5 (b) 13.5
    (c) -27 (d) 27

54. What is the value of $\begin{vmatrix} \sin 10° & -\cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$?

    (a) 0   (b) 1
    (c) -1 (d) 1/2

55. The equation $\tan^2\phi + \tan^6\phi = \tan^3\phi \cdot \sec^2\phi$ is
    (a) identity for only one value of ϕ
    (b) not an identity
    (c) identity for all values of ϕ
    (d) None of the above

56. Which one of the following is not correct ?
    (a) $\sin^{-1}\{\sin(5\pi/4)\} = -\pi/4$
    (b) $\sec^{-1}\{\sec(5\pi/4)\} = 3\pi/4$
    (c) $\tan^{-1}\{\tan(5\pi/4)\} = \pi/4$
    (d) $\csc^{-1}\{\csc(7\pi/4)\} = \pi/4$

57. The foot of a tower of height h m is in a direct line between two observers A and B. If the angles of elevation of the top of the tower as seen from A and B are α and β respectively and if $AB = d$ m, then what is h/d equal to ?
    (a) $\dfrac{\tan(\alpha+\beta)}{(\cot\alpha\cot\beta - 1)}$ (b) $\dfrac{\cot(\alpha+\beta)}{(\cot\alpha\cot\beta - 1)}$
    (c) $\dfrac{\tan(\alpha+\beta)}{(\cot\alpha\cot\beta + 1)}$ (d) $\dfrac{\cot(\alpha+\beta)}{(\cot\alpha\cot\beta + 1)}$

58. If $\sec A + \tan A = p$, then what is the value of $\sin A$?
    (a) $\dfrac{p^2 - 1}{p^2 + 1}$ (b) $\dfrac{p^2 + 1}{p^2 - 1}$
    (c) 1   (d) None of these

59. If $\sin^{-1} x + \sin^{-1} y = \pi/2$ and $\cos^{-1} x - \cos^{-1} y = 0$, then values x and y are respectively
    (a) $\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{2}, \dfrac{1}{2}$
    (c) $\dfrac{1}{2}, -\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$

60. ABC is a triangle in which $AB = 6$ cm, $BC = 8$ cm and $CA = 10$ cm. What is the value of $\cot(A/4)$?
    (a) $\sqrt{5} - 2$ (b) $\sqrt{5} + 2$
    (c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

61. If a set A contains 4 elements, then what is the number of elements in $A \times P(A)$?
    (a) 16   (b) 32   (c) 64   (d) 128

62. If A, B, C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then which one of the following is correct ?
    (a) $A = B$ only   (b) $B = C$ only
    (c) $A = C$ only   (d) $A = B = C$

63. The function $f : R \to R$ defined by $f(x) = (x^2 + 1)^{35}$ for all $x \in R$ is
    (a) one-one but not onto
    (b) onto but not one-one
    (c) neither one-one nor onto
    (d) both one-one and onto

64. The number $(2 + \sqrt{2})^2$ is
    (a) a natural number
    (b) an irrational number
    (c) a rational number
    (d) a whole number

65. If A and B are disjoint sets, then $A \cap (A' \cup B)$ is equal to which one of the following ?
    (a) ϕ
    (b) A
    (c) $A \cup B$
    (d) $A - B'$

66. If A, B, C are three sets, then what is $A - (B - C)$ equal to ?
    (a) $A - (B \cap C)$
    (b) $(A - B) \cup C$
    (c) $(A - B) \cup (A \cap C)$
    (d) $(A - B) \cup (A - C)$

67. If A and B are two subsets of a set X, that what is $A \cap (A \cup B)'$ equal to ?
    (a) A
    (b) B
    (c) ϕ
    (d) A'

68. $f: \{1, 2, 3\} \rightarrow \{4, 5\}$ is not a function if it is defined by which one of the following ?
    (a) $\{(2,4), (3,5), (1,5)\}$
    (b) $\{(1, 4), (2, 4), (3, 4)\}$
    (c) $\{(1, 4), (2, 5), (3, 4)\}$
    (d) $\{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

69. If $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 3), (2, 2), (3, 1) (3, 4), (4, 3), (4, 4)\}$ is a relation on $A \times A$, then which one of the following is correct ?
    (a) R is reflexive
    (b) R is symmetric and transitive
    (c) R is transitive but not reflexive
    (d) R is neither reflexive nor transitive

70. Match List I with List II and select the correct answer using the code given below the lists

	List I		List II
A.	A cube root of unity	1.	$-2(1+i)$
B.	A square root of -1	2.	$2i$
C.	Cube of $1 - i$	3.	$-i$
D.	Square of $1 + i$	4.	$-\frac{1}{2}(1 + i\sqrt{3})$

   Code :.

	A	B	C	D
(a)	4	1	3	2
(b)	2	1	3	4
(c)	4	3	1	2
(d)	2	3	1	4

71. For a positive integer n, what is the value of i^{4n+1} ?
    (a) 1   (b) -1 (c) i (d) $-i$

72. If ω is a complex cube root of unity, then what is the value of $1 - \dfrac{1}{(1 + \omega)} - \dfrac{1}{(1 + \omega^2)}$?
    (a) 1
    (b) 0
    (c) ω
    (d) ω^2

73. What is the 15th term of the series 3, 7, 13, 21, 31, 43, ...?
    (a) 205
    (b) 225
    (c) 238
    (d) 241

74. If the nth term of an arithmetic progression is $2n - 1$, then what is the sum upto n terms ?
    (a) n^2
    (b) $n^2 - 1$
    (c) $n^2 + 1$
    (d) $\frac{1}{2} n(n + 1)$

75. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then what is the value of $b^2 - 4c$?
    (a) 1
    (b) 2
    (c) -2
    (d) 3

76. If $P(A) = 0.8$, $P(B) = 0.9$, $P(AB) = p$, which one of the following is correct ?
    (a) $0.72 \le p \le 0.8$
    (b) $0.7 \le p \le 0.8$
    (c) $0.72 < p < 0.8$
    (d) $0.7 < p < 0.8$

77. What is the value of n for which the numbers $1, 2, 3, \ldots, n$ have variance 2 ?
    (a) 4
    (b) 5
    (c) 6
    (d) 8

78. What is the arithmetic mean of the series ${}^nC_1, {}^nC_2, {}^nC_3, \ldots, {}^nC_n$?
    (a) $(2^n - 1)/n$
    (b) $2^n/(n + 1)$
    (c) $(2^n)/n$
    (d) $2^{(n+1)}/(n + 1)$

79. The average age of 20 students in a class is 15 yr. If the teacher's age is included, the average increases by one. What is the teacher's age ?
    (a) 30 yr
    (b) 21 yr
    (c) 42 yr
    (d) 36 yr

80.

X	1	2	3	4
Frequency	2	3	f	5

   The frequency distribution of a discrete variable X with one missing frequency f is given above. If the arithmetic mean of X is $\dfrac{23}{8}$, what is the value of the missing frequency ?
    (a) 5   (b) 6   (c) 8   (d) 10

81. If the three observations are $3, -6$ and -6, then what is their harmonic mean ?
    (a) 0
    (b) ∞
    (c) $-1/2$
    (d) -3

82. For a set of discrete numbers, three measures of central tendency are given below
    1. Arithmetic mean   2. Median
    3. Geometric mean
    Which of the above measures may not have a meaningful definition ?
    (a) 1 only
    (b) 2 only
    (c) 3 only
    (d) All of them are meaningfully defined

83. Sum of first n natural numbers is given by $\dfrac{n(n+1)}{2}$. What is the geometric mean of the series $1, 2, 4, 8, \ldots, 2^n$?
    (a) 2^n
    (b) $2^{n/2}$
    (c) $2^{1/2}$
    (d) 2^{n-1}

84. Consider the following three methods of collecting data
    (1) collecting data from government offices
    (2) collecting data from public libraries
    (3) collecting data by telephonic interview
    Select the correct answer using the code given below
    (a) All the three methods give secondary data
    (b) 1and 2 give secondary and 3 gives primary data
    (c) 1 and 3 give secondary and 2 gives primary data
    (d) 2 and 3 give secondary and 1 gives primary data

85. If r and s are roots of $x^2 + px + q = 0$, then what is the value of $(1/r^2) + (1/s^2)$?

(a) $p^2 - 4q$

(b) $\dfrac{p^2 - 4q}{2}$

(c) $\dfrac{p^2 - 4q}{q^2}$

(d) $\dfrac{p^2 - 2q}{q^2}$

86. If $(3\vec{a} - \vec{b}) \times (\vec{a} + 3\vec{b}) = k\,\vec{a} \times \vec{b}$, then what is the value of k ?

(a) 10   (b) 5

(c) 8   (d) – 8

87. What is the value of λ if the triangle whose vertices are $\hat{i}, \hat{j}$ and $\hat{i} + \hat{j} + \lambda\hat{k}$ will be right angled ?

(a) 2   (b) 0   (c) –1   (d) 1

88. The scalar triple product $(\vec{A} \times \vec{B}) \cdot \vec{C}$ of three vectors $\vec{A}, \vec{B}, \vec{C}$ determines

(a) Volume of a parallelopiped

(b) Volume of a tetrahedron

(c) Volume of an ellipsoid

(d) None of the above

89. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the value of $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$?

(a) 0   (b) 2

(c) 1   (d) 1 / 2

90. Two forces are equal to $2\vec{OA}$ and $3\vec{BO}$, their resultant being $\lambda\,\vec{OG}$, where G is the point on AB such that $\dfrac{BG}{AG} = -\dfrac{2}{3}$. What is the value of λ ?

(a) 1

(b) –1

(c) 2

(d) None of the above

91. The chance of winning the race of the horse A is 1/5 and that of horse B is 1/6. What is the probability that the race will be won by A or B ?

(a) 1/30   (b) 1/3

(c) 11/30   (d) 1/15

92. What is the probability of two persons being born on the same day (ignoring date) ?

(a) 1/49   (b) 1/365

(c) 1/7   (d) 2/7

93. A coin is tossed. If a head is observed, a number is randomly selected from the set $\{1, 2, 3\}$ and if a tail is observed, a number is randomly selected from the set $\{2, 3, 4, 5\}$. If the selected number be denoted by X, what is the probability that $X = 3$?

(a) 2/7   (b) 1/5   (c) 1/6   (d) 7/24

94. Consider the following statements related to the nature of Bayes' theorem

1. Bayes' theorem is a formula for computation of a conditional probability.

2. Bayes' theorem modifies an assumed probability of an event in the light of a related event which is observed.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

95. The outcomes of an experiment classified as success A or failure A will follow a binomial distribution, if

(a) $P(A) = 1/2$

(b) $P(A) = 0$

(c) $P(A) = 1$

(d) $P(A)$ remains constant in all trials

96. How many words, with or without meaning can be formed by using all the letters of the word 'MACHINE', so that the vowels occurs only the odd positions ?

(a) 1440   (b) 720

(c) 640   (d) 576

97. From 7 men and 4 women a committee of 6 is to be formed such that the committee contains at least two women. What is the number of ways to do this ?

(a) 210   (b) 371

(c) 462   (d) 5544

98. If the AM and GM of two numbers are 5 and 4 respectively, then what is the HM of those numbers ?

(a) $\dfrac{5}{4}$

(b) $\dfrac{16}{5}$

(c) $\dfrac{9}{2}$

(d) 9

99. The arithmetic mean of 4 numbers is 15. The arithmetic mean of another 6 numbers is 12. What is the arithmetic mean of the combined 10 numbers ?

(a) 12.2   (b) 12.8   (c) 13.2   (d) 13.8

100. What is the length of the vector $(1, 1)$?

(a) 0   (b) 1

(c) $\sqrt{2}$ (d) $\dfrac{1}{2}$

101. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle 60° to each other, then which one of the following is correct ?

(a) $|\vec{a} + \vec{b}| < 1$

(b) $|\vec{a} + \vec{b}| > 1$

(c) $|\vec{a} - \vec{b}| < 1$

(d) $|\vec{a} - \vec{b}| > 1$

Directions (Q. Nos. 102-105) : *Each of the next four (4) items consist of two statements, one labelled as the 'Assertiom' (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below*

Code :

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

102. Assertion (A) : The function $f : (1, 2, 3) \to (a, b, c, d)$ defined by $f = \{(1, a), (2, b), (3, c)\}$ has no inverse.

Reason (R) : f is not one-one.

103. ABC is an equilateral triangle inscribed in a circle of centre O and radius 5 cm. Let the diameter through C meet the circle again at

Assertion (A) : $AD \cdot BD < OB \cdot OC$

Reason (R) : $2(AD^2 + BD^2) = CD^2 = 100$ sq cm

104. Assertion (A) : Data collected in decennial censuses are not statistical data.

Reason (R) : Since, no probability is involved in this data collection, it amounts of 100% collection of existing data.

105. Assertion (A) : $y = 2x + 3$ is a one to one real valued function.

Reason (R) : $x_1 \neq x_2$
$\Rightarrow y_1 \neq y_2, y_1 = 2x_1 + 3, y_2 = 2x_2 + 3,$ for any two real x_1 and x_2.

106. What is the value of $\cos 15°$?

(a) $\frac{1}{2}(\sqrt{2 - \sqrt{3}})$　　　　(b) $\frac{1}{2}(\sqrt{2 + \sqrt{3}})$

(c) $\sqrt{2} + \sqrt{3}$　　　　(d) $\sqrt{2} - \sqrt{3}$

107. How many values of θ between $0°$ and $360°$ satisfy $\tan \theta = k \neq 0$, where k is a given number ?

(a) 1　　　　(b) 2

(c) 4　　　　(d) Many

108. If adj $A = \begin{bmatrix} a & 0 \\ -1 & b \end{bmatrix}$ and $ab \neq 0$, then what is the value of $|A^{-1}|$?

(a) 1　　　　(b) ab

(c) $1/\sqrt{ab}$　　　　(d) $1/ab$

109. If $l + m + n = 0$, then the system of equations
$$-2x + y + z = l$$
$$x - 2y + z = m$$
$$x + y - 2z = n$$
has

(a) a trivial solution

(b) no solution

(c) a unique solution

(d) infinitely many solutions

110. If $(a_1/x) + (b_1/y) = c_1, (a_2/x) + (b_2/y) = c_2$,

$\Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$,

$\Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$,

$\Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$,

then (x, y) is equal to which one of the following ?

(a) $(\Delta_2/\Delta_1, \Delta_3/\Delta_1)$　　　　(b) $(\Delta_3/\Delta_1, \Delta_2/\Delta_1)$

(c) $(\Delta_1/\Delta_2, \Delta_1/\Delta_3)$　　　　(d) $(-\Delta_1/\Delta_2, -\Delta_1/\Delta_3)$

111. If $\begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1+p \end{vmatrix} = 20$, then what is the value of p ?

(a) 0　　　　(b) 1

(c) 2　　　　(d) 5

112. If A and B are two matrices such that $AB = A$ and $BA = B$, then which one of the following is correct ?

(a) $(A^T)^2 = A^T$

(b) $(A^T)^2 = B^T$

(c) $(A^T)^2 = (A^{-1})^{-1}$

(d) None of the above

113. If $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, then what is the matrix A ?

(a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -1 \\ 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

114. What is sum to the 100 terms of the series $9 + 99 + 999 + \ldots$?

(a) $\frac{10}{9}(10^{100} - 1) - 100$

(b) $\frac{10}{9}(10^{99} - 1) - 100$

(c) $100(100^{10} - 1)$

(d) $\frac{9}{100}(10^{100} - 1)$

115. What is the minimum value of $2x^2 - 3x + 5$?

(a) 0　　　　(b) 3/4

(c) 31/4　　　　(d) 31/8

116. If α and β are the roots of $x^2 + 4x + 6 = 0$, then what is the value of $\alpha^3 + \beta^3$?

(a) $-2/3$　　　　(b) $2/3$

(c) 4　　　　(d) 8

117. If sum of the roots of $3x^2 + (3p+1)x - (p+5) = 0$ is equal to their product, then what is the value of p ?

(a) 2　　　　(b) 3

(c) 4　　　　(d) 9

118. What is the value of
$^8C_0 - {}^8C_1 + {}^8C_2 - {}^8C_3 + {}^8C_4 - {}^8C_5 + {}^8C_6 - {}^8C_7 + {}^8C_8$

(a) 0　　　　(b) 1

(c) 2　　　　(d) 2^8

119. In how many ways can 3 books on Hindi and 3 books on English be arranged in a row on a shelf, so that not all the Hindi books are together ?

(a) 144　　　　(b) 360

(c) 576　　　　(d) 720

120. If a polygon has 20 diagonals, then what is the number of sides ?

(a) 6　　　　(b) 10

(c) 12　　　　(d) 8

Answers : Mathematics

1	(d)	51	(b)	101	(b)
2	(b)	52	(b)	102	(c)
3	(a)	53	(a)	103	(d)
4	(b)	54	(b)	104	(a)
5	(a)	55	(b)	105	(a)
6	(a)	56	(d)	106	(b)
7	(d)	57	(b)	107	(b)
8	(a)	58	(a)	108	(a)
9	(b)	59	(d)	109	(d)
10	(b)	60	(b)	110	(d)
11	(c)	61	(c)	111	(b)
12	(d)	62	(b)	112	(a)
13	(c)	63	(c)	113	(d)
14	(c)	64	(b)	114	(a)
15	(c)	65	(a)	115	(d)
16	(d)	66	(c)	116	(d)
17	(d)	67	(c)	117	(a)
18	(c)	68	(d)	118	(a)
19	(b)	69	(d)	119	(c)
20	(c)	70	(c)	120	(d)
21	(b)	71	(c)		
22	(d)	72	(b)		
23	(d)	73	(d)		
24	(c)	74	(a)		
25	(b)	75	(a)		
26	(d)	76	(b)		
27	(d)	77	(b)		
28	(c)	78	(a)		
29	(a)	79	(d)		
30	(c)	80	(b)		
31	(a)	81	(b)		
32	(d)	82	(d)		
33	(c)	83	(b)		
34	(d)	84	(b)		
35	(d)	85	(d)		
36	(a)	86	(a)		
37	(b)	87	(b)		
38	(c)	88	(a)		

39	(c)	89	(c)
40	(a)	90	(c)
41	(a)	91	(c)
42	(b)	92	(b)
43	(b)	93	(d)
44	(c)	94	(c)
45	(b)	95	(d)
46	(c)	96	(d)
47	(b)	97	(b)
48	(c)	98	(b)
49	(d)	99	(c)
50	(c)	100	(c)