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UPSC NDA Exam 2007-II Mathematics Solved Paper

Mathematics

1. If $\sin^{-1} x = \tan^{-1} y$, what is the value of $\frac{1}{x^2} - \frac{1}{y^2}$?
(a) 1 (b) – 1
(c) 0 (d) 2

2. If α and β are such that $\tan\alpha = 2\tan\beta$, then what is $\sin(\alpha+\beta)$ equal to ?
(a) 1 (b) $2\sin(\alpha-\beta)$
(c) $\sin(\alpha-\beta)$ (d) $3\sin(\alpha-\beta)$

3. What is the value of
$\cos 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ$?
(a) 1 (b) – 1
(c) 0 (d) 2

4. Let $ABCD$ be a square and let P be a point on AB such that $AP : PB = 1 : 2$. If $\angle APD = \theta$, then what is the value of $\cos\theta$?
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{10}}$ (d) $\frac{2}{\sqrt{5}}$

5. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ satisfies which one of the following polynomial equations ?
(a) $A^2 + 3A + 2I = 0$ (b) $A^2 + 3A - 2I = 0$
(c) $A^2 - 3A - 2I = 0$ (d) $A^2 - 3A + 2I = 0$

6. For how many values of k, will the system of equations $(k+1)x + 8y = 4k$ and $kx + (k+3)y = 3k - 1$, have an infinite number of solutions ?
(a) 1
(b) 2
(c) 3
(d) None of the above

7. For what value of p, is the system of equations :
$$p^3x + (p+1)^3 y = (p+2)^3$$
$$px + (p+1)y = p+2$$
$$x + y = 1$$
consistent ?
(a) $p = 0$ (b) $p = 1$
(c) $p = -1$ (d) For all $p > 1$

8. Among the following equations, which are linear ?
1. $2x + y - z = 5$
2. $\pi x + y - ez = \log 3$
3. $3^x + 2y = 7$
4. $\sin x - y - 5z = 4$
Select the correct answer using the code given below
(a) 1 only
(b) 1 and 2 only
(c) 3 and 4
(d) 1, 2 and 4

9. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then what is the value of x ?
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

10. What is the value of
$\cos\left[\tan^{-1}\left\{\tan\left(\frac{15\pi}{4}\right)\right\}\right]$?
(a) $-\frac{1}{\sqrt{2}}$ (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

11. If $\cos 3A = \frac{1}{2}$, then how many values can $\sin A$ assume?
$(0 < A < 360^\circ)$
(a) 3 (b) 4
(c) 5 (d) 6

12. Consider a circle of radius R. What is the length of a chord which subtends an angle θ at the centre ?
(a) $2R\sin\left(\frac{\theta}{2}\right)$ (b) $2R\sin\theta$
(c) $2R\tan\left(\frac{\theta}{2}\right)$ (d) $2R\tan\theta$

13. Let $A = [a_{ij}]_{n\times n}$ be a matrix and $C = [c_{ij}]_{n\times n}$ be another matrix where c_{ij} is the cofactor of a_{ij}. Then, what is the value of $|AC|$?
(a) $|A|^{n-1}$ (b) $|A|^n$
(c) $|A|^{n+1}$ (d) Zero

14. If ω is the cube root of unity, then what is one root of the equation
$$\begin{vmatrix} x^2 & -2x & -2\omega^2 \\ 2 & \omega & -\omega \\ 0 & \omega & 1 \end{vmatrix} = 0$$?
(a) 1 (b) – 2
(c) 2 (d) ω

15. If $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, then what is A^n equal to ?
(a) $\begin{bmatrix} 2^n & 2^n \\ 2^n & 2^n \end{bmatrix}$ (b) $\begin{bmatrix} 2n & 2n \\ 2n & 2n \end{bmatrix}$
(c) $\begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$ (d) $\begin{bmatrix} 2^{2n+1} & 2^{2n+1} \\ 2^{2n+1} & 2^{2n+1} \end{bmatrix}$

16. If the least number of zeroes in a lower triangular matrix is 10, then what is the order of the matrix ?
(a) 3×3 (b) 4×4 (c) 5×5 (d) 10×10

17. Two angles of a triangle are $\tan^{-1}\frac{1}{2}$ and $\tan^{-1}\frac{1}{3}$. What is the third angle ?
(a) 30° (b) 45° (c) 90° (d) 135°

18. If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$, $z = c\tan\theta$, then what is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ equal to ?
(a) 1 (b) 0 (c) – 1 (d) $a^2 + b^2 -$

19. Let $0^\circ < \theta < 45^\circ$. Which one of the following is correct ?
(a) $\sin^2\theta + \cos^6\theta = \sin^6\theta + \cos^2\theta$
(b) $\text{cosec}^2\theta + \cot^6\theta = \text{cosec}^6\theta + \cot^2\theta$
(c) $\sin^2\theta - \cos^4\theta = \sin^4\theta + \cos^2\theta$
(d) $\text{cosec}^2\theta + \cot^4\theta = \text{cosec}^4\theta - \cot^2\theta$

20. If median of the ΔABC through A is perpendicular to BC, then which one of the following is correct ?
(a) $\tan A + \tan B = 0$ (b) $\tan B - \tan C = 0$
(c) $\tan C + 2\tan A = 0$ (d) $\tan B + \tan C = 0$

21. If $\sin A = \sin B$ and $\cos A = \cos B$, then which one of the following is correct ?
(a) $B = n\pi + A$
(b) $A = 2n\pi - B$
(c) $A = 2n\pi + B$
(d) $B = n\pi - A$ (n is an integer)

22. If α and β are the roots of the equation $x^2 + x + 1 = 0$, then what is the equation whose roots are α^{19} and β^7 ?
(a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$

23. The multiplication of the number $(10101)_2$ by $(1101)_2$ yields which one of the following ?
(a) $(100011001)_2$ (b) $(100010001)_2$
(c) $(110010011)_2$ (d) $(100111001)_2$

24. What is $\frac{(n+2)! + (n+1)(n-1)!}{(n+1)(n-1)!}$ equal to ?
(a) 1
(b) Always an odd integer
(c) A perfect square
(d) None of the above

25. If $1, \omega, \omega^2$ are the three cube roots of unity, then what is $\frac{(a\omega^6 + b\omega^4 + c\omega^2)}{(b + c\omega^{10} + a\omega^8)}$ equal to ?
(a) $\frac{a}{b}$ (b) b
(c) ω (d) ω^2

26. If α and β are the roots of the equation $x^2 + 6x + 1 = 0$, then what is $|\alpha - \beta|$ equal to ?
(a) 6 (b) $3\sqrt{2}$
(c) $4\sqrt{2}$ (d) 12

27. What is the square root of the complex number $-5 + 12i$?
(a) $2 - 3i$ (b) $2 + 3i$
(c) $-2 + 3i$ (d) $\sqrt{-5} + \sqrt{12i}$

28. If x^4 occurs in the rth term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then what is the value of r ?
(a) 4 (b) 8
(c) 9 (d) 10

29. If A and B are two sets satisfying $A - B = B - A$, then which one of the following is correct ?
(a) $A = \phi$ (b) $A \cap B = \phi$
(c) $A = B$ (d) None of these

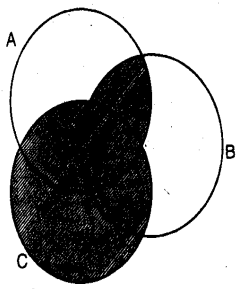
30. Which one of the following is correct ? The real number $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$ is :
(a) an integer
(b) a rational number but not an integer
(c) an irrational number
(d) none of the above

31. If $(A - B) \cup (B - A) = A$ for subsets A and B of the universal set U, then which one of the following is correct ?
(a) B is a proper non-empty subset of A
(b) A and B are non-empty disjoint sets
(c) $B = \phi$
(d) None of the above

32. If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then what is the value of $(A' \cap B')$?
(a) 100 (b) 200
(c) 300 (d) 400

33. If $r^{1/3} + \frac{1}{r^{1/3}} = 3$ for a real number $r \neq 0$, then what is $r + \frac{1}{r}$ equal to ?
(a) 27 (b) 36 (c) 9 (d) 18

34. What does the shaded region in the Venn diagram given above represent ?



(a) $C \cap (A' \cap B')$
(b) $C \cup (C' \cap A \cap B)$
(c) $C \cup (C \cap A) \cup (C \cap B)$
(d) $C \cup (A \setminus B)$

35. If $1, x, y, z, 16$ are in geometric progression, then what is the value of $x + y + z$?
(a) 8 (b) 12
(c) 14 (d) 16

36. The number of rows in a lecture hall equals the number of seats in a row. If the number of rows is doubled and the number of seats in every row in reduced by 10, the number of seats is increased by 300. If x denotes the number of rows in the lecture hall, then what is the value of x ?
(a) 10 (b) 15 (c) 20 (d) 30

37. If $\alpha = \frac{1 + i\sqrt{3}}{2}$, then what is the value of $1 + \alpha^8 + \alpha^{16} + \alpha^{24} + \alpha^{32}$?
(a) 0 (b) 1
(c) ω (d) $-\omega^2$

38. Let N be the set of integers. A relation R on N is defined as $R = \{(x, y) \mid xy > 0, x, y \in N\}$. Then, which one of the following is correct ?
(a) R is symmetric but not reflexive
(b) R is reflexive but not symmetric
(c) R is symmetric and reflexive but not transitive
(d) R is an equivalence relation

39. What is the value of $\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}}$?
(a) $\frac{1}{6}$ (b) $\frac{1}{4}$
(c) 8 (d) 4

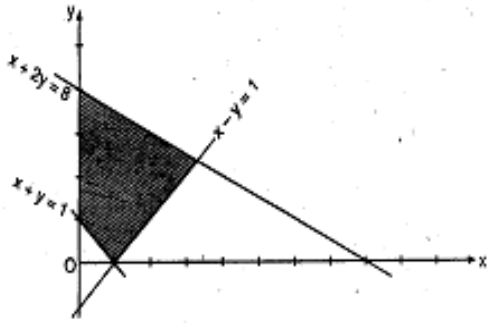
40. If the nth term of an arithmetic progression is $3n+7$, then what is the sum of its first 50 terms ?
(a) 3925 (b) 4100
(c) 4175 (d) 8200

41. If, for positive real numbers x, y, z, the numbers $x+y, 2y$ and $y+z$ are in harmonic progression, then which one of the following is correct ?
(a) x, y, z are in geometric progression
(b) x, y, z are in arithmetic progression
(c) x, y, z are in harmonic progression
(d) None of the above

42. If α, β are the roots of the equation $lx^2 - mx + m = 0$, $l \neq m$, $l \neq 0$, then which one of the following statements is correct ?
(a) $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} - \sqrt{\frac{m}{l}} = 0$
(b) $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{m}{l}} = 0$
(c) $\sqrt{\frac{\alpha+\beta}{\alpha\beta}} - \sqrt{\frac{m}{l}} = 0$
(d) The arithmetic mean of α and β is the same as their geometric mean

43. For what value of k, are the roots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ real and equal ?
(a) $k = 0$ only (b) $k = -3$ only
(c) $k = 0$ or $k = 3$ (d) $k = 0$ or $k = -3$

44. What are the linear constraints for which the shaded area in the above figure is the solution set ?



(a) $x - y \geq 1; x + 2y \leq 8; x + y \geq 1; x, y \geq 0$
(b) $x - y \leq 1; x + 2y \geq 8; x + y \leq 1; x, y \geq 0$
(c) $x - y \leq 1; x + 2y \leq 8; x + y \geq 1; x, y \geq 0$
(d) $x - y \leq 1; x + 2y \leq 8; x + y \leq 1; x, y \geq 0$

45. A 5-digit number divisible by 3 is to be formed using the digits, 0, 1, 2, 3, 4 and 5 without repetition. What is the total number of ways in which this can be done ?
(a) 216 (b) 240
(c) 600 (d) 3125

46. A meeting is to be addressed by 5 speakers A, B, C, D, E. In how many ways can the speakers be ordered, if B must not precede A (immediately or otherwise) ?
(a) 120 (b) 24
(c) 60 (d) $5^4 \times 4$

47. What is the sum of the series $1 + \frac{1}{8} + \frac{1 \cdot 3}{8 \cdot 16} + \frac{1 \cdot 3 \cdot 5}{8 \cdot 16 \cdot 24} + \ldots\ldots \infty$?
(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

48. If roots of an equation $ax^2 + bx + c = 0$ are positive, then which one of the following is correct ?
(a) Signs of a and c should be like
(b) Signs of b and c should be like
(c) Signs of a and b should be like
(d) None of the above

49. What is the geometric mean of the ratio of corresponding terms of two series where G_1 and G_2 are geometric means of the two series ?
(a) $\log G_1 - \log G_2$
(b) $\log G_1 + \log G_2$
(c) $\frac{G_1}{G_2}$
(d) $G_1 G_2$

50. On a railway route there are 20 stations. What is the number of different tickets required in order that it may be possible to travel from every station to every other station ?
(a) 40 (b) 380
(c) 400 (d) 420

51. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is 60°) ?
(a) 20 feet (b) 30 feet
(c) 40 feet (d) $20\sqrt{2}$ feet

52. If $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$, then what is the value of $\text{cosec}^{-1}(\sqrt{5})$?
(a) $\left(\frac{\pi}{2}\right) + \theta$ (b) $\left(\frac{\pi}{2}\right) - \theta$
(c) $\frac{\pi}{2}$ (d) $-\theta$

53. If $\alpha = \frac{\pi}{8}$, what is the value of $\cos\alpha \cos 2\alpha \cos 4\alpha$?
(a) 0 (b) $\frac{1}{4}$
(c) 8 (d) 4

54. What is the value of $\tan^{-1}\left(\frac{m}{n}\right)-\tan^{-1}\left(\frac{m-n}{m+n}\right)$?
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

55. What is the value of $\cot(-870°)$?
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

56. If the inverse of $\begin{bmatrix}1 & p & q\\ 0 & x & 0\\ 0 & 0 & 1\end{bmatrix}$ is $\begin{bmatrix}1 & -p & -q\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ then what is the value of x ?
(a) 1 (b) Zero
(c) -1 (d) $\frac{1}{p}+\frac{1}{q}$

57. If $AB=\begin{bmatrix}4 & 11\\ 4 & 5\end{bmatrix}$ and $A=\begin{bmatrix}3 & 2\\ 1 & 2\end{bmatrix}$, then what is the value of the determinant of the matrix B ?
(a) 4 (b) -6
(c) $-\frac{1}{4}$ (d) -28

58. The determinant
$\begin{vmatrix}a+b+c & a+b & a\\ 4a+3b+2c & 3a+2b & 2a\\ 10a+6b+3c & 6a+3b & 3a\end{vmatrix}$
is independent of which one of the following ?
(a) a and b (b) b and c
(c) a and c (d) All of these

59. Which one of the following is correct ? If the vector $\vec{c}$ is normal to the vectors $\vec{a}$ and $\vec{b}$, then $\vec{c}$ is :
(a) parallel to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$
(b) normal to $\vec{a}-\vec{b}$ and parallel to $\vec{a}+\vec{b}$
(c) normal to $\vec{a}+\vec{b}$ and parallel to $\vec{a}-\vec{b}$
(d) normal to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

60. Which one of the following statements is not correct ?
(a) Vector product is commutative
(b) Vector product is not associative
(c) Vector product is distributive over addition
(d) Scalar product is commutative

61. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+c\hat{k}$ are coplanar vectors, then what is the value of $a+b+c-abc$?
(a) 0 (b) 1
(c) 2 (d) -2

62. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero vectors and $|(\vec{a}\times\vec{b})\cdot\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$, then which one of the following is correct ?
(a) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}\neq 0$
(b) $\vec{a}\cdot\vec{b}=0$ only
(c) $\vec{b}\cdot\vec{c}=0$ only
(d) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

63. $O(0,0)$, $A(0,3)$, $B(4,0)$ are the vertices of triangle OAB. A force $10\hat{i}$ acts at B. What is the magnitude of moment of force about the vertex A ?
(a) 0 (b) 30 unit
(c) 40 unit (d) 50 unit

64. Elements of a population are classified according to the presence or absence of each of 3 attributes A, B and C. What is the number of smallest ultimate classes into which the population is divided ?
(a) 5 (b) 6
(c) 8 (d) 9

65. Students of two schools appeared for a common test carrying 100 marks. The arithmetic means of their marks for school I and II are 82 and 86 respectively. If the number of students of school II is 1.5 times the number of students of school I, what is the arithmetic mean of the marks of all the students of both the schools ?
(a) 84.0
(b) 84.2
(c) 84.4
(d) This cannot be calculated with the given data

66. If AM of numbers $x_1, x_2, \ldots, x_n$ is μ, then what is the AM of the numbers which are increased by $1, 2, 3, \ldots n$ respectively ?
(a) $\mu+\left(\frac{n+1}{2}\right)$ (b) μ
(c) $\mu+\frac{n(n+1)}{2}$ (d) $\mu-\left(\frac{n+1}{2}\right)$

67. In computing a measure of the central tendency for any set of 51 numbers, which one of the following measures is well-defined but uses only very few of the numbers of the set ?
(a) Arithmetic mean (b) Geometric mean
(c) Median (d) Mode

68. Each of A and B tosses two coins. What is the probability that they get equal number of heads ?
(a) $\frac{3}{16}$ (b) $\frac{5}{16}$
(c) $\frac{4}{16}$ (d) $\frac{6}{16}$

69. Examples of some random variables are given below
1. Number of sons among the children of parents with five children
2. Number of sundays in some randomly selected months with 30 days
3. Number of apples in some 3 kg packets, purchased from a retail shop
Which of the above is expected to follow binomial distribution ?
(a) Variable 1 (b) Variable 2
(c) Variable 3 (d) None of these

70. A, B are two events and $\bar{A}$ denotes the complements of A.
Consider the following statements
1. $P(A\cup B)\leq P(B)+P(A)$
2. $P(A)+P(\bar{A}\cup B)\leq 1+P(B)$
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

71. Six text books numbered 1, 2, 3, 4, 5 and 6 are arranged at random. What is the probability that the text books 2 and 3 will occupy consecutive places ?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

72. The data below record the itemwise quarterly expenditure of a private organization :

	Item of expenditure	Amount (in lakh rupees)
1.	Salaries	6.0
2.	TA & DA	4.9
3.	House rent and postage	3.6
4.	All other expenses	5.5
	Total :	20.0

The data is represented by a pie diagram. What is the sectorial angle of the sector with largest area ?
(a) 120° (b) 108°
(c) 100° (d) 90°

73. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+\lambda\hat{k}$, and $(\vec{a}+\vec{b})$ is perpendicular to $\vec{a}-\vec{b}$, then what is the value of λ ?
(a) -2 only (b) ± 2
(c) 3 only (d) ± 3

74. The vectors $\vec{AB}=\vec{c}, \vec{BC}=\vec{a}, \vec{CA}=\vec{b}$, are the sides of a triangle ABC. Which of the following vectors represents (s) the median $\vec{AD}$?
1. $\frac{1}{2}\vec{a}+\vec{c}$ 2. $-\frac{1}{2}\vec{b}+\frac{1}{2}\vec{c}$
3. $\frac{1}{2}\vec{a}+\vec{b}$
Select the correct answer using the code given below
(a) 1 and 2 (b) 1 and 3
(c) 1 only (d) 2 only

75. What is the area enclosed by the curve $2x^2+y^2=1$?
(a) 2π (b) π
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{\sqrt{2}}$

76. A radioactive element disintegrates at a rate proportional to the quantity of substance Q present at any time t. What is the differential equation of the disintegration ?
(a) $\frac{dQ}{dt}=-Q$ (b) $\frac{dQ}{dt}=-kQ, k<0$
(c) $\frac{dQ}{dt}=-kQ, k>0$ (d) $\frac{dQ}{dt}=Q$

77. What is the solution of the differential equation $(x+y)(dx-dy)=dx+dy$?
(a) $2\log(x+y)=c(y-x)$ (b) $(y-x)+\log(x+y)=c$
(c) $\left(\frac{y}{x}\right)+\left[\log\left(\frac{y}{x}\right)\right]=c$ (d) None of these

78. What is the only solution of the initial value problem $y'=t(1+y), y(0)=0$?
(a) $y=-1+e^{t^2/2}$ (b) $y=1+e^{t^2/2}$
(c) $y=-t$ (d) $y=t$

79. What is the differential equation of the curve $y=ax^2+bx$?
(a) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$
(b) $x^2\frac{d^2y}{dx^2}-y\left(\frac{dy}{dx}\right)^2+2=0$
(c) $(1-x^2)\frac{d^2y}{dx^2}-\left(y\frac{dy}{dx}\right)^2=0$
(d) None of the above

80. What is the degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=k\frac{d^2y}{dx^2}$?
(a) 4 (b) 3 (c) 2 (d) 1

81. What is the value of integral $I=\int_0^1 x(1-x)^9\,dx$?
(a) $\frac{1}{110}$ (b) $\frac{1}{111}$ (c) $\frac{1}{112}$ (d) $\frac{1}{119}$

82. What is the value of $\int_{-1}^{1} x|x|\,dx$?
(a) 2 (b) 1 (c) $\frac{1}{4}$ (d) 0

83. What is the value of $\int_0^{\pi/2}\cos^8 x\,dx$?
(a) $\frac{35\pi}{256}$ (b) $\frac{70}{256}$ (c) $\frac{16}{35}$ (d) $\frac{8\pi}{35}$

84. What is $\int_a^b \frac{\log x}{x}dx$ equal to ?
(a) $(1/2)\log(ab)\cdot\log(b/a)$
(b) $\log b/\log a$
(c) $\log(b/a)$
(d) $(1/2)\log[(a+b)/ab]$

85. What is the value of $\int\frac{e^x(1+x)}{\sin^2(xe^x)}dx$?
(a) $-e^x\cot x+c$ (b) $\cos^2(xe^x)+c$
(c) $\log\sin(xe^x)+c$ (d) $-\cot(xe^x)+c$

86. Which one of the following is correct ? The function $f(x)=(x-1)e^x+1$ is
(a) negative for all $x>0$ (b) positive for all $x>0$
(c) increasing for all x (d) decreasing for all x

87. If a differentiable function f defined for $x>0$ satisfies the relation $f(x^2)=x^3$, $x>0$, then what is the value of $f'(4)$?
(a) 1 (b) 2 (c) 3 (d) 4

88. The motion of a particle is described as $s=2-3t+4t^3$. What is the acceleration of the particle at the point where its velocity is zero ?
(a) 0 (b) 4 unit
(c) 8 unit (d) 12 unit

89. The coordinates of P and Q are $(-3, 4)$ and $(2, 1)$, respectively. If PQ is extended to R such that $PR=2QR$, then what are the coordinates of R ?
(a) (3, 7) (b) (2, 4) (c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$ (d) (7, −2)

90. If the points with the coordinates (a, ma), $\{b, (m+1)b\}$, $\{c, (m+2)c\}$ are collinear, then which one of the following is correct ?
(a) a, b, c are in arithmetic progression for all m
(b) a, b, c are in geometric progression for all m
(c) a, b, c are in harmonic progression for all m
(d) a, b, c are in arithmetic progression only for $m = 1$

91. Which one of the following points on the line $2x - 3y = 5$ is equidistant from (1, 2) and (3, 4) ?
(a) (7, 3) (b) (4, 1)
(c) (1, −1) (d) (−2, −3)

92. What is the equation of circle which touches the lines $x = 0, y = 0$ and $x = 2$?
(a) $x^2 + y^2 + 2x + 2y + 1 = 0$ (b) $x^2 + y^2 - 4x - 4y + 1 = 0$
(c) $x^2 + y^2 - 2x - 2y + 1 = 0$ (d) None of these

93. In how many points do the ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$ and the circle $x^2 + y^2 = 9$ intersect ?
(a) One
(b) Two
(c) Four
(d) None of the above

94. If the foci of the conics $\frac{x^2}{a^2} + \frac{y^2}{7} = 1$ and $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ were to coincide, then what is the value of a ?
(a) 2 (b) 3 (c) 4 (d) 16

95. What is the ratio in which the line joining the points (2, 4, 5) and (3, 5, −4) is internally divided by the xy-plane?
(a) 5 : 4 (b) 3 : 4
(c) 1 : 2 (d) 7 : 5

96. A line makes angles θ, ϕ and ψ with x, y, z axes respectively. Consider the following
1. $\sin^2\theta + \sin^2\phi = \cos^2\psi$
2. $\cos^2\theta + \cos^2\phi = \sin^2\psi$
3. $\sin^2\theta + \cos^2\phi = \cos^2\psi$
Which of the above is/are correct ?
(a) 1 only (b) 2 only
(c) 3 only (d) 2 and 3

97. What is the equation of the plane passing through (x_1, y_1, z_1) and normal to the line with $< a, b, c >$ as direction ratios ?
(a) $ax + by + cz = ax_1 + by_1 + cz_1$
(b) $a(x + x_1) + b(y + y_1) + c(c + z_1) = 0$
(c) $ax + by + cz = 0$
(d) $ax + by + cz = x_1 + y_1 + z_1 = 0$

98. What are the direction cosines of the line represented by $3x + y + 2z = 7$, $x + 2y + 3z = 5$?
(a) (−1, −7, 5) (b) (−1, 7, 5)
(c) $\left(-\frac{1}{\sqrt{75}}, -\frac{7}{\sqrt{75}}, \frac{5}{\sqrt{75}}\right)$ (d) $\left(-\frac{1}{\sqrt{75}}, \frac{7}{\sqrt{75}}, \frac{5}{\sqrt{75}}\right)$

99. The equation of a sphere is $x^2 + y^2 + z^2 - 10z = 0$. If one end point of a diameter of the sphere is (−3, −4, 5), what is the other end point ?
(a) (−3, −4, −5) (b) (3, 4, 5)
(c) (3, 4, −5) (d) (−3, 4, −5)

100. Given, $f(x) = x + \frac{1}{x}$, then what is $f^2(x)$ equal to ?
(a) $\frac{x^2+1}{x} + \frac{x}{x^2+1}$ (b) $(x + 1/x)^2$
(c) $x^4 + (1/x^4)$ (d) $x^2 + (1/x^2)$

101. Which one of the following is correct ? If $4 < x^2 < 9$, then
(a) $2 < x < 3$ only (b) $-3 < x < -2$ only
(c) $2 < x < 3, -3 < x < -2$ (d) None of these

102. If $f(x) = \begin{cases} 1, & x \text{ is a rational number} \\ 0, & x \text{ is an irrational number,} \end{cases}$ what is/are the value(s) of $(fof)(\sqrt{3})$?
(a) 0 (b) 1
(c) Both 0 and 1 (d) None of these

103. A function f is defined as follows
$f(x) = x^p \cos\left(\frac{1}{x}\right), x \neq 0$
$f(0) = 0$
What conditions should be imposed on p so that f may be continuous at $x = 0$?
(a) $p = 0$ (b) $p > 0$
(c) $p < 0$ (d) No value of p

104. What is the derivative of $\tan^{-1}\left(\frac{\sqrt{x} - x}{1 + x^{3/2}}\right)$ at $x = 1$?
(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{2}$ (d) 1

105. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, what is $\frac{dy}{dx}$ equal to ?
(a) $-\frac{1}{1+x}$ (b) $-\frac{1}{(1+x)^2}$
(c) $\frac{1}{(1+x)^2}$ (d) $\frac{\sqrt{x}}{\sqrt{1+x}}$

106. What is the product of two parts of 20, such that the product of one part and the cube of the other is maximum?
(a) 75 (b) 91
(c) 84 (d) 96

107. What is the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$?
(a) 1 (b) 2
(c) 5 (d) −23

108. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{\ldots\infty}}}}$, then what is $f'(x)$ equal to ?
(a) $\frac{1}{1 - 2f(x)}$ (b) $\frac{1}{2f(x) - 1}$
(c) $\frac{1}{1 + 2f(x)}$ (d) $\frac{1}{2 + f(x)}$

Directions : The next four (4) items consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below:

Codes :
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and r are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

109. While constructing the cumulative frequency column of a frequency distribution, it is noticed that these cumulative frequecies are in arithmetic progression.
Assertion (A) : All the class frequencies are equal.
Reason (R) : When all the class frequencies are equal, the cumulative frequencies are in arithmetic progression.

110. **Assertion (A) :** If events, A, B, C, D are mutually exhaustive, then $(A \cup B \cup C)^C = D$.
Reason (R) : $(A \cup B \cup C)^C = D$ implies if any element is excluded from the sets A, B and C, then it is included in D.

111. **Assertion (A) :** If two triangles with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3) satisfy the relation
$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the triangles are congruent.
Reason (R) : For the given triangles satisfying the above relation implies that the triangles have equal area.

112. **Assertion (A) :** $0.3 + 0.03 + 0.003 + \ldots\ldots = \frac{1}{3}$.
Reason (R) : For each (+)ve integer n, let $a_n = a + nd$, a and d are real numbers. Then, $a_1 + \ldots\ldots\ldots + a_n = \frac{n}{2}[2a + (n+1)d]$.

113. Which one of the following is correct ? If the positive numbers a, b, c, d are in AP, then bcd, cda, dab, abc
(a) are in AP
(b) are in GP
(c) are in HP
(d) are in none of the above progressions

114. What is the value of $9^{1/3} \cdot 9^{1/19} \cdot 9^{1/27} \ldots\ldots \infty$?
(a) 9 (b) 3 (c) $9^{1/3}$ (d) 1

115. If a, b, c, d are in harmonical progression such that $a > d$, then which one of the following is correct ?
(a) $a + c = b + d$ (b) $a + c > b + d$
(c) $ac = bd$ (d) $ab = cd$

116. After simplification, what is the number of terms in the expansion of $[(3x + y)^5]^4 - [(3x - y)^4]^5$?
(a) 4 (b) 5 (c) 10 (d) 11

117. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then what are the roots of the equation $cx^2 + bx + a = 0$?
(a) $\beta, \frac{1}{\alpha}$ (b) $\alpha, \frac{1}{\beta}$ (c) $-\alpha, -\beta$ (d) $\frac{1}{\alpha}, \frac{1}{\beta}$

118. If x and y are real numbers such that $x > y$ and $|x| > |y|$, then which one of the following is correct?
(a) $x > 0$ (b) $y > 0$
(c) $y < 0$ (d) $x < 0$

119. For what value (s) of x is $\log_{10}\{999 + \sqrt{x^2 - 3x + 3}\} = 3$?
(a) 0 (b) 1 only
(c) 2 only (d) 1, 2

120. If in a frequency distribution table with 12 classes, the width of each class is 2.5 and the lowest class boundary is 6.1, then what is the upper class boundary of the highest class ?
(a) 30.1 (b) 27.6
(c) 30.6 (d) 36.1

Ans: Mathematics

1	(a)
2	(d)
3	(c)
4	(a)
5	(c)
6	(b)
7	(c)
8	(b)
9	(b)
10	(a)
11	(b)
12	(a)
13	(c)
14	(b)
15	(c)
16	(b)
17	(d)
18	(a)
19	(d)
20	(b)
21	(a)
22	(d)
23	(b)
24	(c)
25	(c)
26	(c)
27	(b)
28	(c)
29	(c)
30	(b)
31	(c)
32	(c)
33	(d)
34	(c)
35	(c)
36	(d)
37	(d)
38	(d)

51	(b)
52	(b)
53	(a)
54	(c)
55	(a)
56	(a)
57	(b)
58	(b)
59	(d)
60	(a)
61	(c)
62	(d)
63	(b)
64	(d)
65	(c)
66	(a)
67	(d)
68	(b)
69	(b)
70	(c)
71	(d)
72	(b)
73	(b)
74	(c)
75	(d)
76	(b)
77	(b)
78	(a)
79	(a)
80	(c)
81	(a)
82	(d)
83	(a)
84	(a)
85	(d)
86	(c)
87	(c)
88	(c)

101	(c)
102	(b)
103	(b)
104	(a)
105	(b)
106	(a)
107	(c)
108	(b)
109	(a)
110	(a)
111	(a)
112	(d)
113	(c)
114	(b)
115	(b)
116	(c)
117	(d)
118	(a)
119	(d)
120	(d)

39	(d)	89	(d)
40	(c)	90	(c)
41	(a)	91	(b)
42	(a)	92	(c)
43	(c)	93	(d)
44	(c)	94	(c)
45	(a)	95	(a)
46	(b)	96	(b)
47	(a)	97	(a)
48	(a)	98	(d)
49	(c)	99	(b)
50	(b)	100	(b)