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UPSC NDA Exam 2007-I Mathematics Solved Paper

Mathematics

1. For any two vectors $\vec{a}$ and $\vec{b}$, consider the following statements :

1. $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}| \Leftrightarrow \vec{a}, \vec{b}$ are orthogonal.
2. $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}| \Leftrightarrow \vec{a}, \vec{b}$ are orthogonal
3. $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2 \Leftrightarrow \vec{a}, \vec{b}$ are orthogonal.

Which of the above statements are correct ?

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

2. Two vectors $2\hat{i}+m\hat{j}-3n\hat{k}$ and $5\hat{i}+3m\hat{j}+n\hat{k}$ are such that their magnitudes are respectively $\sqrt{14}$ and $\sqrt{35}$, where m, n are integers. Which one of the following is correct ?

(a) m takes 1 value, n takes 1 value
(b) m takes 1 value, n takes 2 values
(c) m takes 2 values, n takes 1 value
(d) m takes 2 values, n takes 2 values

3. If $\int_{\ln 2}^{x}(e^x-1)^{-1}\,dx=\ln\frac{3}{2}$, then what is the value of x ?

(a) e^2 (b) $\frac{1}{e}$
(c) $\ln 4$ (d) 1

4. If $\int_{-3}^{2} f(x)\,dx=\frac{7}{3}$ and $\int_{-3}^{9} f(x)\,dx=-\frac{5}{6}$, then what is the value of $\int_{2}^{9} f(x)\,dx$?

(a) $-\frac{19}{6}$ (b) $\frac{19}{6}$
(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

5. What is the value of $\int\frac{dx}{(x^2+a^2)(x^2+b^2)}$?

(a) $\frac{[\{\tan^{-1}(x/a)\}/a-\{\tan^{-1}(x/b)\}/b]}{(a^2+b^2)}+c$

(b) $\frac{[\{\tan^{-1}(x/a)\}/a+\{\tan^{-1}(x/b)\}/b]}{(a^2+b^2)}+c$

(c) $\frac{[\{\tan^{-1}(x/a)\}/a+\{\tan^{-1}(x/b)\}/b]}{(b^2-a^2)}+c$

(d) $\frac{[\{\tan^{-1}(x/a)\}/a+\{\tan^{-1}(x/b)\}/b]}{(b^2-a^2)}+c$

6. What is the equation of the curve passing through the origin and satisfying the differential equation $dy=(y\tan x+\sec x)\,dx$?

(a) $y=x\cos x$ (b) $y\cos x=x$
(c) $xy=\cos x$ (d) $y\sin x=x$

7. What is the solution of the differential equation $\frac{dy}{dx}=\sec(x+y)$?

(a) $y+\tan(x+y)=c$ (b) $y-\tan\left\{\frac{(x+y)}{2}\right\}=c$
(c) $y+\tan\left\{\frac{(x+y)}{2}\right\}=c$ (d) $y+\tan\left\{\frac{(x-y)}{2}\right\}=c$

8. For what value of k, does the differential equation $\frac{dy}{dx}=ky$ represent the law of natural decay ?

(a) -5 (b) 0
(c) 0.01 (d) $(10)^{-1}$

9. What is/are the critical point(s) of the function $f(x)=x^{2/3}(5-2x)$ on the interval $[-1, 2]$?

(a) 1 only (b) 0, 1
(c) $\frac{3}{2}$ only (d) $0, \frac{3}{2}$

10. Match List I with List II and select the correct answer using the code given below the lists :

List I	List II
A. $f(x)=\cos x$	1. The graph cuts y-axis in infinite number of points
B. $f(x)=\ln x$	2. The graph cuts x-axis in two points
C. $f(x)=x^2-5x+4$	3. The graph cuts y-axis in only one point
D. $f(x)=e^x$	4. The graph cuts x-axis in only one point
	5. The graph cuts x-axis in infinite number of points

Code :

	A	B	C	D
(a)	1	4	5	3
(b)	1	3	5	4
(c)	5	4	2	3
(d)	5	3	2	4

11. If $f(x)=(x+1)^{\cot x}$ is continuous at $x=0$, then what is $f(0)$ equal to ?

(a) 1 (b) e
(c) $\frac{1}{e}$ (d) e^2

12. What is the solution of the differential equation $(x+y)(dx-dy)=dx+dy$?

(a) $x+y+\ln(x+y)=c$ (b) $x-y+\ln(x+y)=c$
(c) $y-x+\ln(x+y)=c$ (d) $y-x-\ln(x-y)=c$

13. If $x+y=12$, what is the maximum value of xy ?

(a) 25 (b) 36
(c) 49 (d) 64

14. The lower 24 m portion of a 50 m tall tower is painted green and the remaining portion red. What is the distance of a point on the ground from the base of the tower where the two different portions of the tower subtend equal angles ?

(a) 60 m (b) 72 m
(c) 90 m (d) 120 m

15. If A, B and C are angles of a triangle such that $\tan A = 1$, $\tan B = 2$, then what is the value of $\tan C$?
(a) 0 (b) 1
(c) 2 (d) 3

16. What is the acute angle between the lines $Ax + By = A + B$ and $A(x - y) + B(x + y) = 2B$?
(a) 45° (b) $\tan^{-1}\left(\frac{A}{\sqrt{A^2+B^2}}\right)$
(c) $\tan^{-1}\left(\frac{B}{\sqrt{A^2+B^2}}\right)$ (d) 60°

17. If the sum of the squares of the distances of the point (x, y, z) from the points $(a, 0, 0)$ and $(-a, 0, 0)$ is $2c^2$, then which one of the following is correct ?
(a) $x^2 + a^2 = 2c^2 - y^2 - z^2$ (b) $x^2 + a^2 = c^2 - y^2 - z^2$
(c) $x^2 - a^2 = c^2 - y^2 - z^2$ (d) $x^2 + a^2 = c^2 + y^2 + z^2$

18. Under which one of the following conditions does the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meet the x-axis in two points on opposite sides of the origin ?
(a) $c > 0$ (b) $c < 0$
(c) $c = 0$ (d) $c \leq 0$

19. If p be the length of the perpendicular from the origin on the straight line $x + 2by = 2p$, then what is the value of b ?
(a) $\frac{1}{p}$ (b) p
(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

20. What is the equation to the parabola , whose vertex and focus are on the x-axis at distances a and b from the origin respectively ? $(b > a > 0)$
(a) $y^2 = 8(b - a)(x - a)$ (b) $y^2 = 4(b + a)(x - a)$
(c) $y^2 = 4(b - a)(x + a)$ (d) $y^2 = 4(b - a)(x - a)$

21. What is the equation of a circle, whose centre lies on the x-axis at a distance h from the origin and the circle passes through the origin ?
(a) $x^2 + y^2 - 2hx = 0$ (b) $x^2 + y^2 - 2hx + h^2 = 0$
(c) $x^2 + y^2 + 2hxy = 0$ (d) $x^2 + y^2 - h^2 = 0$

22. Which one of the following is correct ?
The three planes $2x + 3y - z - 2 = 0$,
$3x + 3y + z - 4 = 0$, $x - y + 2z - 5 = 0$ intersect
(a) at a point (b) at two points
(c) at three points (d) in a line

23. If the eccentricity and length of latusrectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis ?
(a) $\frac{7}{2}$ unit (b) 12 unit
(c) $\frac{15}{2}$ unit (d) $\frac{15}{4}$ unit

24. In what ratio does the line $y - x + 2 = 0$ cut the line joining $(3, -1)$ and $(8, 9)$?
(a) 2 : 3 (b) 3 : 2
(c) 3 : − 2 (d) 1 : 2

Directions : The following four (4) items consists of two statements, one labelled as the **'Assertion (A)'** and the other as **'Reason (R)'**. You are to examine these two statements carefully and select the answers to these items using the codes given below :
Codes :
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is ***not*** the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

25. Assertion (A) : If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $(A + B)^2 = A^2 + B^2 + 2AB$.
Reason (R) : In the above $AB = BA$

26. Assertion (A) : If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}$ and $B = \begin{pmatrix} \cos\alpha & \cos\alpha \\ \sin\alpha & \sin\alpha \end{pmatrix}$, then $AB \neq I$.
Reason (R) : The product of two matrices can never be equal to an identity matrix.

27. Assertion (A) : $\int_0^{\pi} \sin^7 x\, dx = 2\int_0^{\pi/2} \sin^7 x\, dx$
Reason (R) : $\sin^7 x$ is an odd function

28. Assertion (A) : If $< l, m, n >$ are direction cosines of a line, there can be a line whose direction cosines are $\left\langle \sqrt{\frac{l^2+m^2}{2}}, \sqrt{\frac{m^2+n^2}{2}}, \sqrt{\frac{n^2+l^2}{2}} \right\rangle$
Reason (R) : The sum of direction cosines of a line is unity.

29. What is one of the values of $\sqrt{i} + \sqrt{-i}$?
(a) $\sqrt{2}$ (b) 0
(c) $\pm\frac{1+i}{\sqrt{2}}$ (d) $\pm\frac{1-i}{\sqrt{2}}$

30. If the equation $x^2 + k^2 = 2(k + 1)x$ has equal roots, then what is the value of k ?
(a) $-\frac{1}{3}$ (b) $-\frac{1}{2}$
(c) 0 (d) 1

31. If $x = a^{1/3} - a^{-1/3}$, then what is $x^3 + 3x$ equal to ?
(a) Zero (b) $a + \left(\frac{1}{a}\right)$
(c) $a - \left(\frac{1}{a}\right)$ (d) $a^3 + \left(\frac{1}{a^3}\right)$

32. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$ then what is $(x + y + z)^3$ equal to ?
(a) 1 (b) 3
(c) $3xyz$ (d) $27xyz$

33. a, b, c are in G.P. with $1 < a < b < n$, and $n > 1$ is an integer. $\log_a n$, $\log_b n$, $\log_c n$ form a sequence. This sequence is which one of the following ?
(a) Harmonic progression (b) Arithmetic progression
(c) Geometric progression (d) None of these

34. What is the value of
$$\frac{\operatorname{cosec}(\pi+\theta)\cot\{(9\pi/2)-\theta\}\operatorname{cosec}^2(2\pi-\theta)}{\cot(2\pi-\theta)\sec^2(\pi-\theta)\sec\{(3\pi/2)+\theta\}}?$$
(a) 0 (b) 1
(c) –1 (d) ∞

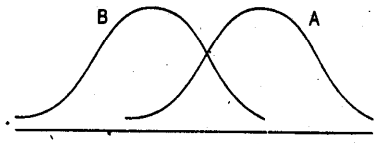
35. If $3^{(x-1)}+3^{(x+1)}=30$, then what is the value of $3^{(x+2)}+3^x$?
(a) 30 (b) 60
(c) 81 (d) 90

36. Let $f:[-100\pi, 1000\pi]\to[-1,1]$ be defined by $f(\theta)=\sin\theta$. Then what is the number of values of $\theta\in[-100\pi, 1000\pi]$ for which $f(\theta)=0$?
(a) 1000 (b) 1101
(c) 1100 (d) 1110

37. Which one of the following is the plane containing the line $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{5}$ and parallel to z-axis?
(a) $2x-3y=0$ (b) $5x-2z=0$
(c) $5y-3z=0$ (d) $3x-2y=0$

38. Which one of the following statement is ***not*** correct?
(a) Median divides distributions into two equal subgroups
(b) The third quartile is the same as the 75th percentile
(c) The 5th decile is the same as the 50th percentile
(d) The 50th decile is the same as the 5th percentile

39. The mean weight of all the students in a certain class is 60 kg. The mean weight of the boys from the class is 70 g, while that of the girls is 55 kg. What is the ratio of number of boys to that of girls ?
(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

40.


Frequency curves for the distribution of blood pressure readings of certain athletes before exercise (A) and after exercise (B) are plotted together as shown in the figure above. From the frequency curves, which one of the following can be concluded?
(a) Both distributions are identical
(b) Both distributions have the same mean value
(c) Both distributions have the same mean value but different variance
(d) Both distributions have the same variance but different mean values

41. If the slopes of the line of regression of Y and X and of X and Y are 30°and 60° respectively, then $r(X, Y)$ is :
(a) –1 (b) 1
(c) $\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$

42. What is the probability of getting five heads and seven tails in 12 flips of a balanced coin?
(a) C (12, 5)/(2^5) (b) C (12, 5)/(2^7)
(c) C (12, 5)/(2^{12}) (d) C (12, 7)/(2^6)

43. The points (2, – 2), (8, 4), (4, 6) and (–1, 1) in order are the vertices of which one of the following quadrilaterals?
(a) Square
(b) Rhombus
(c) Rectangle (but not square)
(d) Trapezium

44. If p be the length of the perpendicular from the origin on the straight line $ax+by=p$ and $b=\frac{\sqrt{3}}{2}$, then what is the angle between the perpendicular and the positive direction of x-axis ?
(a) 30° (b) 45°
(c) 60° (d) 90°

45. The straight line $ax+by+c=0$ and the coordinate axes form an isosceles triangle under which one of the following conditions?
(a) $|a|=|b|$ (b) $|a|=|c|$
(c) $|b|=|c|$ (d) None of these

46. What is the centre of the sphere $ax^2+by^2+cz^2-6x=0$, if the radius is 1 unit?
(a) (0, 0, 0)
(b) (1, 0, 0)
(c) (3, 0, 0)
(d) Cannot be determined as values of a, b, c are unknown

47. Under what condition do $\left\langle\frac{1}{\sqrt{2}}, \frac{1}{2}, k\right\rangle$ represent direction cosines of a line?
(a) $k=\frac{1}{2}$ (b) $k=-\frac{1}{2}$
(c) $k=\pm\frac{1}{2}$ (d) k can take any value

48. What is the value of $\lim_{x\to\infty}\left(\frac{x-2}{x+2}\right)^{x+2}$?
(a) 0 (b) e^4
(c) e^{-2} (d) e^{-4}

49. If $f(x)=(x+1)\tan^{-1}(e^{-2x})$, then what is the value of $f'(0)$?
(a) $\left(\frac{\pi}{4}\right)+1$ (b) $\left(\frac{\pi}{4}\right)-1$
(c) $\left(\frac{\pi}{2}\right)+1$ (d) $\frac{\pi}{4}$

50. What is the x-coordinate of the point on the curve $f(x)=\sqrt{x}(7x-6)$, where the tangent is parallel to x-axis?
(a) $-\frac{1}{3}$ (b) $\frac{2}{7}$
(c) $\frac{6}{7}$ (d) $\frac{1}{3}$

51. If $\sin x\cos y=\frac{1}{2}$, then what is the value of $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$?
(a) –4 (b) –2
(c) – 6 (d) 0

52. What is the interval in which the function $f(x)=\sqrt{9-x^2}$ is increasing? $(f(x)>0)$

(a) $0<x<3$ (b) $-3<x<0$

(c) $0<x<9$ (d) $-3<x<3$

53. If the derivative of the function

$$f(x)=\begin{cases} ax^2+b & x<-1 \\ bx^2+ax+4 & x\geq -1 \end{cases}$$

is everywhere continuous, then what are the values of a and b?

(a) $a=2, b=3$ (b) $a=3, b=2$

(c) $a=-2, b=-3$ (d) $a=-3, b=-2$

54. A wire 34 cm long is to be bent in the form of a quadrilateral of which each angle is 90°. What is the maximum area which can be enclosed inside the quadrilateral?

(a) 68 cm^2 (b) 70 cm^2

(c) 71.25 cm^2 (d) 72.25 cm^2

55. What is the degree of the differential equation $k\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^3\right]^{3/2}$, where k is a constant?

(a) 1 (b) 2 (c) 3 (d) 4

56. Under which one of the following conditions does the solution of $\frac{dy}{dx}=\frac{ax+b}{cy+d}$ represent a parabola?

(a) $a=0, c=0$ (b) $a=1, b=2, c\neq 0$

(c) $a=0, c\neq 0, b\neq 0$ (d) $a=1, c=1$

57. What is the value of $\int(\sqrt{x}+x)^{-1}\,dx$?

(a) $\ln(x+\sqrt{x})+c$ (b) $2\ln(1+\sqrt{x})+c$

(c) $2\ln(x+\sqrt{x})+c$ (d) $2\ln(x-\sqrt{x})+c$

58. If $f(x)$ is differentiable everywhere, then which one of the following is correct?

(a) $|f|$ is differentiable everywhere

(b) $|f|^2$ is differentiable everywhere

(c) $f|f|$ is not differentiable at some points

(d) None of the above

59. Let $f: R\to R$ be defined as $f(x)=ax^2+bx+c$, a, b, c being fixed non-zero real numbers. Which one of the following statements is correct, in general?

(a) If $b^2-4ac>0$, then $f^{-1}(0)$ does not contain 0

(b) If $b^2-4ac<0$, then $f^{-1}(0)$ must contain 0

(c) If $b^2-4ac>0$, then $f^{-1}(0)$ may contain 0

(d) If $b^2-4ac<0$, then $f^{-1}(0)$ may contain 0

60. What is the coefficient of x^5 in the expansion $(1-2x+3x^2-4x^3+\cdots\infty)^{-5}$?

(a) $(10!)/(5!)^2$ (b) 5^{-5}

(c) 5^5 (d) $10!/\{(6!)(4!)\}$

61. If $\frac{x-a}{b+c}+\frac{x-b}{c+a}+\frac{x-c}{c+b}=3$, then what is the value of x?

(a) 0 (b) 1

(c) $a+b+c$ (d) abc

62. What is the value of $[(-1+i\sqrt{3})/2]^{10}+[(-1-i\sqrt{3})/2]^{10}$?

(a) 1 (b) −1 (c) 2 (d) 0

63. How many real values of x satisfy the equation $|x|+|x-1|=1$?

(a) 1 (b) 2

(c) Infinite (d) No value of x

64. If A is any 2×2 matrix such that

$$\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}A=\begin{bmatrix}-1 & 0\\ 6 & 3\end{bmatrix}$$

then what is A equal to?

(a) $\begin{bmatrix}-5 & 1\\ -2 & 2\end{bmatrix}$ (b) $\begin{bmatrix}-5 & -2\\ 1 & 2\end{bmatrix}$

(c) $\begin{bmatrix}-5 & -2\\ 2 & 1\end{bmatrix}$ (d) $\begin{bmatrix}5 & 2\\ -2 & -1\end{bmatrix}$

65. If α, β are the roots of $ax^2+2bx+c=0$ and $\alpha+\delta, \beta+\delta$ are the roots of $Ax^2+2Bx+C=0$, then what is $(b^2-ac)/(B^2-AC)$ equal to?

(a) $(b/B)^2$ (b) $(a/A)^2$

(c) $(a^2b^2)/(A^2B^2)$ (d) $(ab)/(AB)$

66. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$?

(a) $C(12, 7)\,x^3\,y^{-3}$

(b) $C(12, 6)\,x^{-3}\,y^3$

(c) $C(12, 7)\,x^{-3}\,y^3$

(d) $C(12, 6)\,x^3\,y^{-3}$

67. If α, β are the roots of the equation $ax^2+bx+c=0$, then what is the value of $(a\alpha+b)^{-1}+(a\beta+b)^{-1}$?

(a) $a/(bc)$ (b) $b/(ac)$

(c) $-b/(ac)$ (d) $-a/(bc)$

68. What is the number of digits in the numeral form of 8^{17}? (Given $\log_{10}2=0.3010$)

(a) 51 (b) 16

(c) 15 (d) 14

69. What is the sum of the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots$?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

70. If b_1, b_2, b_3 are three consecutive terms of an arithmetic progression with common difference $d>0$, then what is the value of d for which $b_3^2=b_2b_3+b_1d+2$?

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) 2

71. If α, β are the roots of the equation $x^2-2x-1=0$, then what is the value of $\alpha^2\beta^{-2}+\alpha^{-2}\beta^2$?

(a) −2 (b) 0

(c) 30 (d) 34

72. Which one of the following values of x, y satisfies the in equation $2x+3y\leq 6; x\geq 0, y\geq 0$?

(a) $x=0, y=3$ (b) $x=1, y=2$

(c) $x=1, y=1$ (d) $x=4, y=0$

73. What is the value of x at the intersection of $y = \frac{8}{(x^2+4)}$ and $x + y = 2$?

(a) 0 (b) 1

(c) 2 (d) –1

74. If A is a 3×3 matrix such that $|A| = 4$, then what is A(adj A) equal to?

(a) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

(b) $\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$

(c) $\begin{bmatrix}16&0&0\\0&16&0\\0&0&16\end{bmatrix}$

(d) Cannot be determined, as data is insufficient

75. If $A = \begin{bmatrix}x&x^2&1+x^2\\y&y^2&1+y^2\\z&z^2&1+z^2\end{bmatrix}$ where x, y, z are distinct. What is $|A|$?

(a) 0 (b) $x^2y - y^2x + xyz$

(c) $(x-y)(y-z)(z-x)$ (d) xyz

76. Two vectors $\vec{a}$ and $\vec{b}$ are non-zero and non-collinear. What is the value of x for which the vectors $\vec{p} = (x-2)\vec{a} + \vec{b}$ and $\vec{q} = (x+1)\vec{a} - \vec{b}$ are collinear?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) 2

77. If $\vec{a}$ and $\vec{b}$ are position vectors of the points A and B respectively, then what is the position vector of a point C on AB produced such that $\vec{AC} = 2\vec{AB}$?

(a) $2\vec{a} - \vec{b}$ (b) $2\vec{b} - \vec{a}$

(c) $\vec{a} - 2\vec{b}$ (d) $\vec{a} - \vec{b}$

78. If $|\vec{a}| = 3, |\vec{b}| = 4$, then for what value of λ is $(\vec{a} + \lambda\vec{b})$ perpendicular to $(\vec{a} - \lambda\vec{b})$?

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{9}{16}$ (d) $\frac{3}{5}$

79. What is the magnitude of the moment of the couple consisting of the force $\vec{F} = 3\hat{i} + 2\hat{j} - \hat{k}$ acting through the point $\hat{i} - \hat{j} + \hat{k}$ and $-\vec{F}$ acting through the point $2\hat{i} - 3\hat{j} - \hat{k}$?

(a) $2\sqrt{5}$ (b) $3\sqrt{5}$

(c) $5\sqrt{5}$ (d) $7\sqrt{5}$

80. Let $\vec{a} = 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{j} + 3\hat{k}$ and $\vec{c} = -3\hat{i} + 3\hat{j} + \hat{k}$. Let $\hat{n}$ be a unit vector such that $\vec{a}\cdot\hat{n} = \vec{b}\cdot\hat{n} = 0$. What is the value of $\vec{c}\cdot\hat{n}$?

(a) 1 (b) $\sqrt{19}$

(c) 3 (d) –3

81. Let $\vec{u} = \hat{i} - \hat{j}$, $\vec{v} = 2\hat{i} + 5\hat{j}$, $\vec{w} = 4\hat{i} + 3\hat{j}$ and $\vec{p} = \vec{u} + \vec{v} + \vec{w}$. Which one of the following is correct?

(a) $-3\vec{u} + 2\vec{v} = \vec{p}$ (b) $3\vec{u} - 2\vec{v} = \vec{p}$

(c) $3\vec{u} + 2\vec{v} = \vec{p}$ (d) $-3\vec{u} - 2\vec{v} = \vec{p}$

82. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle of 30° to each other, then which one of the following is correct ?

(a) $|\vec{a} + \vec{b}| > 1$ (b) $1 < |\vec{a} + \vec{b}| < 2$

(c) $|\vec{a} + \vec{b}| = 2$ (d) $|\vec{a} + \vec{b}| > 2$

83. If you want to measure the intelligence of a group of students, which one of the following measures will be more suitable ?

(a) Arithmetic mean

(b) Mode

(c) Median

(d) Geometric mean

84. In a lottery, 16 tickets are sold and 4 prizes are awarded. If a person buys 4 tickets, what is the probability of his winning a prize ?

(a) $\frac{4}{16^4}$ (b) $\frac{175}{256}$

(c) $\frac{1}{4}$ (d) $\frac{81}{256}$

85. If A and B are any two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, where $\bar{A}$ stands for the complementary event of A, then what is $P(B)$?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{9}$ (d) $\frac{2}{9}$

86. In a binomial distribution, the mean is 4 and the variance is 3. What is the mode ?

(a) 6 (b) 5

(c) 4 (d) 3

87. In how many ways can be letters of the word 'CABLE' be arranged so that the vowels should always occupy odd positions?

(a) 12 (b) 18

(c) 24 (d) 36

88. If X is changed to $a + hU$ and Y to $b + kV$, then which one of the following is the correct relation between the regression coefficients b_{XY} and b_{UV}?

(a) $h\,b_{XY} = k\,b_{UV}$ (b) $k\,b_{XY} = h\,b_{UV}$

(c) $b_{XY} = b_{UV}$ (d) $k^2 b_{XY} = h^2 b_{UV}$

89. A card is drawn from a pack of 52 cards and a gambler bets that it is a spade or an ace. Which one of the following are the odds against his winning this bet?
(a) 13 to 4 (b) 4 to 13
(c) 9 to 4 (d) 4 to 9

90. A can hit a target 4 times in 5 shots;
B can hit a target 3 times in 4 shots;
C can hit a target 2 times in 3 shots;
All the three fire a shot each. What is the probability that two shots are at least hit?
(a) 1/6 (b) 3/5
(c) 5/6 (d) 1/3

91. A box contains 10 identical electronic components of which 4 are defective. If 3 components are selected at random from the box in succession, without replacing the units already drawn, what is the probability that two of the selected components are defective?
(a) 1/5 (b) 5/24
(c) 3/10 (d) 1/40

92. For non-empty subsets A, B and C of a set X such that $A \cup B = B \cap C$, which one of the following is the strongest inference that can be derived?
(a) $A = B = C$ (b) $A \subseteq B = C$
(c) $A = B \subseteq C$ (d) $A \subseteq B \subseteq C$

93. If μ is the universal set and P is a subset of μ, then what is $P \cap \{(P - \mu) \cup (\mu - P)\}$ equal to?
(a) ϕ (b) P' (c) μ (d) P

94. Let μ = the set of all triangles, P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right-angled triangles. What do the sets $P \cap Q$ and $R - P$ represents respectively?
(a) The set of isosceles triangles; the set of non-isosceles right-angled triangles
(b) The set of isosceles triangles; the set of right angled triangles
(c) The set of equilateral triangles; the set of right-angled triangles
(d) The set of isosceles triangles, the set of equilateral triangles

95. Consider the following statements :
For non empty sets A, B and C
1. $A - (B - C) = (A - B) \cup C$
2. $A - (B \cup C) = (A - B) - C$
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

96. If the roots of the equations $x^2 - (a-1)x + (a+b) = 0$ and $ax^2 - 2x + b = 0$ are identical, then what are the values of a and b?
(a) $a = 2, b = 4$ (b) $a = 2, b = -4$
(c) $a = 1, b = \frac{1}{2}$ (d) $a = -1, b = -\frac{1}{2}$

97. A relation R is defined over the set of non-negative integers as $x\,R\,y \Rightarrow x^2 + y^2 = 36$. What is R?
(a) $\{(0, 6)\}$
(b) $\{(6, 0), (\sqrt{11}, 5), (3, 3, \sqrt{3})\}$
(c) $\{(6, 0), (0, 6)\}$
(d) $\{(\sqrt{11}, 5), (2, 4\sqrt{2}), (5\sqrt{11}), (4\sqrt{2}, 2)\}$

98. Consider the following statements :
1. Parallelism of lines is an equivalence relation.
2, $x\,R\,y$, if x is a father of y, is an equivalence relation.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

99. If ω denotes the cube root of unity, then what is the real root of the equation $x^3 - 27 = 0$?
(a) 3ω (b) $3\omega^2$
(c) -3ω (d) $3\omega^3$

100. Let O be the origin and point A be represented by z. If OA is rotated through an angle $\pi/2$ in the anticlockwise direction keeping the length of OA same, then what represents the new point?
(a) $-iz$ (b) $|z|i$
(c) iz (d) z

101. Which one of the following binary numbers is the prime number?
(a) 111101 (b) 111010
(c) 111111 (d) 100011

102. What is the product of the binary numbers 1001.01 and 11.1?
(a) 101110.011 (b) 100000.011
(c) 101110.101 (d) 100000.101

103. Under which of the following condition(s), will the matrix $A = \begin{bmatrix} 0 & 0 & q \\ 2 & 5 & 1 \\ 8 & p & p \end{bmatrix}$ be singular?
1. $q = 0$
2. $p = 0$
3. $p = 20$
Select the correct answer using the code given below :
(a) 1 and 2 (b) 3 only
(c) 1 and 3 (d) 1 or 3

104. Consider the following statements :
1. If $\det A = 0$, then $\det(\text{adj } A) = 0$
2. If A is non-singular, then $\det(A^{-1}) = (\det A)^{-1}$
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

105. Let A be an $m \times n$ matrix. Under which one of the following conditions does A^{-1} exist?
(a) $m = n$ only
(b) $m = n$ and $\det A \neq 0$
(c) $m = n$ and $\det A = 0$
(d) $m \neq n$

106. Let A and B be two matrices of order $n \times n$. Let A be non-singular and B be singular. Consider the following :
1. AB is singular
2. AB is non-singular
3. $A^{-1}B$ is singular
4. $A^{-1}B$ is non singular
Which of the above is/are correct?
(a) 1 and 3 (b) 2 and 4
(c) 1 only (d) 3 only

107. Let A be a square matrix of order $n \times n$ where $n \geq 2$. Let B be a matrix obtained from A with first and second rows interchanged. Then which one of the following is correct?

(a) $\det A = \det B$ (b) $\det A = -\det B$

(c) $A = B$ (d) $A = -B$

108. What should be the value of k so that the system of linear equations $x - y + 2z = 0$, $kx - y + z = 0$, $3x + y - 3z = 0$ does not possess a unique solution?

(a) 0 (b) 3

(c) 4 (d) 5

109. If $-x^2 + 3x + 4 > 0$, then which one of the following is correct ?

(a) $x \in (-1, 4)$

(b) $x \in [-1, 4]$

(c) $x \in (-\infty, -1) \cup (4, \infty)$

(d) $x \in (-\infty, -1] \cup [4, \infty)$

110. What is the value of

$\sin(A + B)\sin(A - B) + \sin(B + C)\sin(B - C) + \sin(C + A)\sin(C - A)$?

(a) 0. (b) $\sin A + \sin B + \sin C$

(c) $\cos A + \cos B + \cos C$ (d) 1

111. Given that $\tan\alpha = m/(m + 1)$, $\tan\beta = 1/(2m + 1)$, then what is the value of $\alpha + \beta$?

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

112. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then $x^2 + y^2 + z^2$ is independent of which of the following ?

(a) r only (b) r, ϕ

(c) θ, ϕ (d) r, θ

113. What is the minimum value of $\cos\theta + \cos 2\theta$?

(a) -2 (b) $-\frac{9}{8}$

(c) 0 (d) $-\frac{9}{16}$

114. If $3\tan\theta + 4 = 0$, where $(\pi/2) < \theta < \pi$, then what is the value of $2\cot\theta - 5\cos\theta + \sin\theta$?

(a) $-\frac{53}{10}$ (b) $\frac{7}{10}$

(c) $\frac{23}{10}$ (d) $\frac{37}{10}$

115. What is $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$, where $x > 0$, equal to?

(a) $\sqrt{\frac{(x^2+1)}{(x^2+2)}}$ (b) $\sqrt{\frac{(x^2+2)}{(x^2+1)}}$

(c) $\frac{(x^2+1)}{(x^2+1)}$ (d) $\frac{(x^2+2)}{(x^2+1)}$

116. What is the value of $\operatorname{cosec}(13\pi/12)$?

(a) $\sqrt{6} + \sqrt{2}$ (b) $-\sqrt{6} + \sqrt{2}$

(c) $\sqrt{6} - \sqrt{2}$ (d) $-\sqrt{6} - \sqrt{2}$

117. What is the value of

$(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)(\cot\theta + \tan\theta)$?

(a) 1 (b) 2

(c) $\sin\theta$ (d) $\cos\theta$

118. If $\alpha + \beta = \frac{\pi}{2}$ and $\beta + \gamma = \alpha$; then which one of the following is correct?

(a) $2\tan\beta + \tan\gamma = \tan\alpha$ (b) $\tan\beta + 2\tan\gamma = \tan\alpha$

(c) $\tan\beta + \tan\gamma = \tan\alpha$ (d) $2(\tan\beta + \tan\gamma) = \tan\alpha$

119. What is the value of $\frac{(\cos 10^\circ + \sin 20^\circ)}{(\cos 20 - \sin 10^\circ)}$?

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

120. In a triangle ABC, if $a = 2b$ and $A = 3B$ then which one of the following is correct?

(a) The triangle is obtuse-angled

(b) The triangle is acute-angled but not right-angled

(c) The triangle is right-angled

(d) The triangle is isosceles, but not obtuse-angled

Answer: Mathematics

1	(b)
2	(a)
3	(c)
4	(a)
5	(d)
6	(a)
7	(b)
8	(a)
9	(a)
10	(c)
11	(b)
12	(c)
13	(b)
14	(d)
15	(d)
16	(a)
17	(b)
18	(b)
19	(d)
20	(d)
21	(a)
22	(d)
23	(c)
24	(a)
25	(a)
26	(d)
27	(b)
28	(c)
29	(a)
30	(b)
31	(c)
32	(d)
33	(a)
34	(b)
35	(d)
36	(c)
37	(d)
38	(d)

51	(a)
52	(b)
53	(a)
54	(d)
55	(b)
56	(b)
57	(c)
58	(d)
59	(a)
60	(a)
61	(c)
62	(b)
63	(b)
64	(c)
65	(b)
66	(d)
67	(b)
68	(b)
69	(d)
70	(c)
71	(d)
72	(c)
73	(a)
74	(b)
75	(a)
76	(b)
77	(b)
78	(a)
79	(c)
80	(d)
81	(c)
82	(b)
83	(b)
84	(c)
85	(b)
86	(c)
87	(b)
88	(b)

101	(a)
102	(b)
103	(d)
104	(c)
105	(b)
106	(b)
107	(b)
108	(d)
109	(a)
110	(a)
111	(b)
112	(c)
113	(b)
114	(c)
115	(a)
116	(d)
117	(a)
118	(b)
119	(c)
120	(c)

39	(b)	89	(c)
40	(d)	90	(c)
41	(c)	91	(c)
42	(c)	92	(d)
43	(d)	93	(a)
44	(c)	94	(a)
45	(a)	95	(b)
46	(d)	96	(b)
47	(c)	97	(c)
48	(d)	98	(a)
49	(b)	99	(d)
50	(b)	100	(c)