JAGRAN

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UPSC NDA Exam 2006- II Mathematics
Solved Paper

Directions :

The following four (4) items consist of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the code given below :

Code :

(a) Both *A* and *R* are individually true and *R* is the correct explanation of *A*

(b) Both *A* and *R* are individually true but *R* is not the correct explanation of *A*

(c) *A* is true but *R* is false

(d) *A* is false but *R* is true

1. *Assertion (A)* : The number of triangles that can be formed by joining the mid-points of any three adjacent faces of a cube is 20.
   Reason (R) : If there are *n* points on a plane and none of them are collinear, then the number of triangles that can be formed is $C(n, 3)$.

2. *Assertion (A)* : The number of selections of 20 distinct things taken 8 at a time is same as that taken 12 at a time.
   Reason (R) : $C(n, r) = C(n, s)$, if $n = r + s$.

3. *Assertion (A)* : $1/8, \log_a^2 a, \log_a a^2$ are in GP but not in AP
   Reason (R) : *x, y, z* are in AP as well as in GP if $x = y = z$.

4. *Assertion (A)* : If $Z_1 = 3 + \sqrt{-4}$, and $Z_2 = 3 + \sqrt{-25}$, Z_1/Z_2 is a complex number.
   Reason (R) : If Z_1, Z_2 are complex numbers, then Z_1/Z_2 is always a complex number.

5. An aircraft has three engines *A, B* and *C*. The aircraft crashes if all the three engines fail. The probabilities of failure are 0.03, 0.02 and 0.05 for engines. *A, B* and *C* respectively. What is the probability that the aircraft will not crash ?
   (a) 0.00003   (b) 0.90
   (c) 0.99997   (d) 0.90307

6. A coin is tossed three times. What is the probability of getting head and tail (HTH) or tail and head (THT) alternately ?
   (a) 1/4   (b) 1/5
   (c) 1/6   (d) 1/8

7. The probability that a student passes in mathematics is 4/9 and that he passes in physics is 2/5. Assuming that passing in mathematics and physics are independent of each other, what is the probability that he passes in mathematics but fails in physics ?
   (a) 4/15   (b) 8/45
   (c) 26/45   (d) 19/45

8. From a pack of 52 cards, two cards are drawn, the first being replaced before the second is drawn. What is the probability that the first is a diamond and the second is a king ?
   (a) 1/4
   (b) 4/13
   (c) 1/52
   (d) 4/15

9. What is the probability of having a knave and a queen when two cards are drawn from a pack of 52 cards ?
   (a) 16/663   (b) 2/663
   (c) 4/663   (d) 8/663

10. Consider the following statement :
    "The mean of a binomial distribution is 3 and variance is 4."
    Which of the following is correct regarding this statement?
    (a) It is always true
    (b) It is sometimes true
    (c) It is never true
    (d) No conclusion can be drawn

11. In throwing of two dice, what is the number of exhaustive events ?
    (a) 6   (b) 12
    (c) 36   (d) 18

12. If from the point of intersection of two ogives, a perpendicular is drawn on the *x*-axis, what does the *x*-coordinate give ?
    (a) Arithmetic Mean   (b) Mode
    (c) Median   (d) Geometric Mean

13. The marks scored by two students *A* and *B* in six subjects are given below :

A	71	56	45	89	54	44
B	55	74	83	54	38	52

Which one of the following statements is correct ?
(a) The average scores of *A* and *B* are same but *A* is consistent
(b) The average scores of *A* and *B* are not same but *A* is consistent
(c) The average scores of *A* and *B* are same but *B* is consistent
(d) The average scores of *A* and *B* are not same but *B* is consistent

14. Consider the following statements :
    1. Equations $ax + by + cz + d = 0$, $a'x + b'y + c'z + d' = 0$ represent a straight line.
    2. Equation of the form
    $$\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$$
    represent a straight line passing through the point (α, β, γ) and having direction ratios proportional to *l, m, n*.
    Which of the statements given above is/are correct ?
    (a) 1 only   (b) 2 only
    (c) Both 1 and 2   (d) Neither 1 nor 2

15. If the centre of the sphere $ax^2 + by^2 + cz^2 - 2x + 4y + 2z - 3 = 0$ is $(1/2, -1, -1/2)$, what is the value of *b* ?
   (a) 1   (b) –1
    (c) 2   (d) –2

16. What is the length of the perpendicular from the origin to the plane $ax + by + \sqrt{2\,ab}\, z = 1$?
    (a) 1/(*ab*)   (b) 1/(*a* + *b*)
    (c) *a* + *b*   (d) *ab*

17. What is the equivalent definition of the function given by

$$f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases} ?$$

(a) $f(x) = |x|$
(b) $f(x) = 2x$
(c) $f(x) = |x| + x$
(d) $f(x) = 2|x|$

18. If $f : R \to R^+$ such that $f(x) = (1/3)^x$, then what is $f^{-1}(x)$ equal to ?

(a) $(1/3)^x$
(b) 3^x
(c) $\log_{1/3} x$
(d) $\log_x (1/3)$

19. What is the value of $\lim\limits_{x \to 0} \dfrac{x \sin 5x}{\sin^2 4x}$?

(a) 0
(b) 5/4
(c) 5/16
(d) 25/4

20. If $f(x) = (1 + x)^{5/x}$ is continuous at $x = 0$, then what is the value of $f(0)$?

(a) 0
(b) 1
(c) ∞
(d) e^5

21. Consider the following statements :
1. The function $f(x) = $ greatest integer $\leq x$, $x \in R$ is a continuous function.
2. All trigonometric functions are continuous on R.
Which of the statements given above is/are correct ?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

22. Which one of the following statements is not correct ?
(a) The derivative of $f(x)$ at $x = a$ is the slope of the graph of $f(x)$ at the point $[a, f(a)]$
(b) $f(x)$ has a positive derivative at $x = a$ means $f(x)$ increases as x increases from 'a'
(c) The sum of two differentiable functions is differentiable
(d) If a function is continuous at a point, it is also differentiable at the same point

23. What is the derivative of $f(x) = \sqrt{1 - x^2}$ with respect to $g(x) = \sin^{-1} x$, where $|x| \neq 1$?
(a) x
(b) $-x$
(c) $\dfrac{x}{1 - x^2}$
(d) $-\dfrac{x}{1 - x^2}$

24. What is the derivative of $(\log_{\tan x} \cot x) (\log_{\cot x} \tan x)^{-1}$ at $x = \pi/4$?
(a) -1
(b) 0
(c) 1
(d) 1/2

25. Which one of the following statements is correct in respect of the curve $4y - x^2 - 8 = 0$?
(a) The curve is increasing in $(-4, 4)$
(b) The curve is increasing in $(-4, 0)$
(c) The curve is increasing in $(0, 4)$
(d) The curve is decreasing in $(-4, 4)$

26. What is the minimum value of $px + qy$ $(p > 0, q > 0)$ when $xy = r^2$?
(a) $2r \sqrt{pq}$
(b) $2pq \sqrt{r}$
(c) $-2r \sqrt{pq}$
(d) $2rpq$

27. Let $45° \leq \theta < 90°$. If $\tan \theta + \cot \theta = (\tan \theta)^i + (\cot \theta)^i$ for some $i \geq 2$, then what is the value of $\sin \theta + \cos \theta$?
(a) $\sqrt{2}$
(b) $\dfrac{1}{\sqrt{2}}$
(c) $\dfrac{(\sqrt{3} + 1)}{2}$
(d) $\dfrac{2}{(\sqrt{3} + 1)}$

28. Given that $\tan \theta = m \neq 0$, $\tan 2\theta = n \neq 0$ and $\tan \theta + \tan 2\theta = \tan 3\theta$, then which one of the following is correct ?
(a) $m = n$
(b) $m + n = 1$
(c) $m + n = 0$
(d) $mn = -1$

29. Let A and B be obtuse angles such that $\sin A = 4/5$ and $\cos B = -12/13$. What is the value of $\sin (A + B)$?
(a) $-63/65$
(b) $-33/65$
(c) $33/65$
(d) $63/65$

30. For what values of a does the equation $\cos 2x + a \sin x = 2a - 7$ possess a real solution ?
(a) $a < 2$
(b) $a \geq 8$
(c) $a > 8$
(d) a is any integer < -2

31. If $\tan^2 B = \dfrac{1 - \sin A}{1 + \sin A}$, then what is the value of $A + 2B$?
(a) $\pi/2$
(b) $\pi/3$
(c) $\pi/4$
(d) $\pi/6$

32. What are the values of (x, y) satisfying the simultaneous equations $\sin^{-1} x + \sin^{-1} y = 2\pi/3$ and $\cos^{-1} x - \cos^{-1} y = \pi/3$?
(a) $(0, 1)$
(b) $(1/2, 1)$
(c) $(1, 1/2)$
(d) $(\sqrt{3}/2, 1)$

33. Given that $\cos 20° - \sin 20° = p$, then what is the value of $\sin 40°$?
(a) $1 - p^2$
(b) $1 + p^2$
(c) p^2
(d) $p^2 - 1$

34. If the perimeter of a triangle ABC is 30 cm, then what is the value of $a \cos^2 (C/2) + c \cos^2 (A/2)$?
(a) 15 cm
(b) 10 cm
(c) 15/2 cm
(d) 13 cm

35. In ΔABC, if $\angle A : \angle B : \angle C = 1 : 2 : 3$, then what is $BC : CA : AB$?
(a) $1 : 2 : 3$
(b) $1 : \sqrt{3} : 2$
(c) $2 : \sqrt{3} : 1$
(d) $\sqrt{3} : 1 : 2$

36. The angle of elevation of the top of a pillar of height h at a point on the ground at a distance x from the pillar is 30°. On walking a distance 'd' towards the pillar the angle of elevation becomes 60°. Then, which one of the following is correct ?
(a) $x = d + h$
(b) $x = 3d/2$
(c) $x = 5d/4$
(d) $x = 2d$

37. If $\sin \theta$ and $\cos \theta$ are the roots of $ax^2 + bx + c = 0$, then constants a, b, c will satisfy which one of the following conditions ?
(a) $a^2 + b^2 + 2ac = 0$
(b) $a^2 + b^2 - 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$
(d) $-a^2 + b^2 + 2ac = 0$

38. The angle of elevation of the top of a tower EF (F being the foot of the tower) as seen from a point A which is on the same level as F, is α. On advancing towards the foot of the tower the angle of elevation of the top of the tower as seen from a point B such that $AB = x$, is β. If $BF = y$, h is the height of the tower and $\alpha + \beta = \pi/2$, then which one of the following is correct ?
(a) $h^2 = x^2 + xy$
(b) $h = y^2 + xy^2$
(c) $h^2 = y^2 + xy$
(d) $h = y + x^2 y$

39. What is the image of the point (2, 3) in the line $y = -x$?
    (a) (-3, -2)
    (b) (-3, 2)
    (c) (-2, -3)
    (d) (3, 2)

40. If $x + 1, 4x + 1$, and $8x + 1$ are in geometric progression, then what is the non-trivial value of x ?
    (a) -1
    (b) 1
    (c) $\frac{1}{8}$
    (d) $\frac{1}{4}$

41. The maximum three digit integer in the decimal system will be represented in the binary system by which one of the following ?
    (a) 1111110001
    (b) 1111111110
    (c) 1111100111
    (d) 1111000111

42. What is the difference between the smallest five digit binary integer and the largest four digit binary integer?
    (a) The smallest four digit binary integer
    (b) The smallest one digit binary integer
    (c) The greatest one digit binary integer
    (d) The greatest three digit binary integer

43. Let $z = i^3 (1 + i)$ be a complex number. What is its argument ?
    (a) π
    (b) $\frac{\pi}{4}$
    (c) $-\frac{\pi}{4}$
    (d) $\frac{5\pi}{4}$

44. The equation $(a^2 + b^2) x^2 - 2b(a + c) x + (b^2 + c^2) = 0$ has equal roots. Which one of the following is correct about a, b and c ?
    (a) They are in AP
    (b) They are in GP
    (c) They are in HP
    (d) They are neither in AP, nor in GP, nor in HP

45. Let z_1 and z_2 be two non-zero complex numbers such that

    $$|z_1| = |z_2| = \left| \frac{1}{z_1} + \frac{1}{z_2} \right| = 2.$$

    What is the value of $|z_1 + z_2|$?
    (a) 8
    (b) 4
    (c) 2
    (d) 1

46. If $F(n)$ denotes the set of all divisors of n except 1, what is the least value of y satisfying $[F(20) \cap F(16)] \subseteq F(y)$?
    (a) 1
    (b) 2
    (c) 4
    (d) 8

47. On the set Z of integers, relation R is defined as "$a R b \Leftrightarrow a + 2b$ is an integral multiple of 3". Which one of the following statements is correct for R ?
    (a) R is only reflexive
    (b) R is only symmetric
    (c) R is only transitive
    (d) R is an equivalence relation

48. For non-empty sets A, B and C, the following two statements are given :
    Statement $P : A \cap (B \cup C) = (A \cap B) \cup C$
    Statement $Q : C$ is a subset of A
    Which one of the following is correct ?
    (a) $P \Leftarrow Q$
    (b) $P \Leftrightarrow Q$
    (c) $P \Rightarrow Q$
    (d) Nothing can be said about the correctness of the above three with certainty

49. If $X = \{x : x > 0, x^2 < 0\}$, and
    $Y = \{$flower, Churchill, moon, Kargil$\}$, then which one of the following is a correct statement ?
    (a) X is well defined but Y is not a well defined set
    (b) Y is well defined but X is not a well defined set
    (c) Both X and Y are well defined sets
    (d) Neither X nor Y is a well defined set

50. Consider the following for any three non-empty sets A, B and C :
    1. $A - (B \cup C) = (A - B) \cup (A - C)$
    2. $A - B = A - (A \cap B)$
    3. $A = (A \cap B) \cup (A - B)$
    Which of the above is/are correct ?
    (a) Only 1
    (b) 2 and 3
    (c) 1 and 2
    (d) 1 and 3

51. If p^{th} term of an AP is q, and its qth term is p, then what is the common difference ?
    (a) -1
    (b) 0
    (c) 2
    (d) 1

52. Consider the following statements :
    There are infinitely many rational numbers between two distinct
    1. integers.
    2. rational numbers.
    3. real numbers.
    Which of the statements above are correct ?
    (a) Only 1 and 2
    (b) Only 2 and 3
    (c) Only 1 and 3
    (d) 1, 2 and 3

53. What does the shaded region represent in the figure given below ?
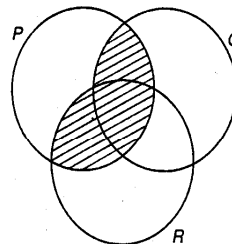
    (a) $(P \cup Q) - (P \cap Q)$
    (b) $P \cap (Q \cup R)$
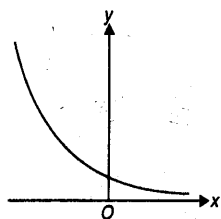   (c) $(P \cap Q) \cap (P \cap R)$
    (d) $(P \cap Q) \cup (P \cap R)$

54. If $\lim\limits_{x \to a} \left[\dfrac{f(x)}{g(x)} \right]$ exists, then which one of the following is correct ?
    (a) Both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ must exist
    (b) $\lim\limits_{x \to a} f(x)$ need not exist but $\lim\limits_{x \to a} g(x)$ must exist
    (c) Both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ need not exist
    (d) $\lim\limits_{x \to a} f(x)$ must exist but $\lim\limits_{x \to a} g(x)$ need not exist

55. If $f(x) = \begin{cases} mx + 1 & x \le \pi/2 \\ \sin x + n & x > \pi/2 \end{cases}$
    is continuous at $x = \pi/2$, then which one of the following is correct ?
    (a) $m = 1, n = 0$
    (b) $m = \dfrac{n\pi}{2} + 1$
    (c) $n = m \left(\dfrac{\pi}{2} \right)$
    (d) $m = n = \dfrac{\pi}{2}$

56.

The above curve shows the graph of a^x under which one of the following conditions ?
(a) $a \geq 1$ (b) $a > 1$
(c) $0 < a \leq 1$ (d) $0 < a < 1$

57. What is the derivative of $\cos^{-1}\left(\dfrac{2\cos x + 3\sin x}{\sqrt{13}}\right)$?

(a) $\dfrac{1}{\sqrt{1-x^2}}$ (b) $-\dfrac{1}{\sqrt{1-x^2}}$

(c) 0   (d) 1

58. What is the derivative of $f(x) = \dfrac{7x}{(2x-1)(x+3)}$?

(a) $-\dfrac{3}{(x+3)^2} - \dfrac{2}{(2x-1)^2}$ (b) $-\dfrac{3}{(x+3)^2} - \dfrac{1}{(2x-1)^2}$

(c) $\dfrac{3}{(x+3)^2} + \dfrac{1}{(2x-1)^2}$ (d) $\dfrac{3}{(x+3)^2} + \dfrac{2}{(2x-1)^2}$

59. What is the value of $\int_0^1 (x-1)\, e^{-x}\, dx$?

(a) 0   (b) e
(c) $1/e$ (d) $-1/e$

60. If $\int \sec x \csc x\, dx = \log |g(x)| + c$, then what is $g(x)$ equal to ?
(a) $\sin x \cos x$ (b) $\sec^2 x$
(c) $\tan x$ (d) $\log |\tan x|$

61. What are the order and degree respectively of the differential equation $y = x\dfrac{dy}{dx} + \dfrac{dx}{dy}$?
(a) 1, 1   (b) 1, 2
(c) 2, 1   (d) 2, 2

62. What is the solution of $y' = 1 + x + y^2 + xy^2,\, y(0) = 0$?

(a) $y = \tan^2\left(\dfrac{x^2}{2} + x\right)$ (b) $y = \tan^2(x^2 + x)$

(c) $y = \tan(x^2 + x)$ (d) $y = \tan\left(\dfrac{x^2}{2} + x\right)$

63. If $a^x = b, b^y = c, c^z = a$, then what is the value of $\dfrac{1}{(xy + yz + zx)}\left(\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}\right)$?
(a) 0   (b) abc
(c) 1   (d) -1

64. If $2^x = 3^y = 12^z$, then what is $(x + 2y)/(xy)$ equal to ?
(a) z (b) $1/z$
(c) $2z$ (d) $z/2$

65. If $a^2 + b^2 + c^2 = 0$, then what is

$$\dfrac{(a^4 - b^4)^3 + (b^4 - c^4)^3 + (c^4 - a^4)^3}{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}$$

equal to ?
(a) $a^2 b^2 c^2$ (b) $-a^2 b^2 c^2$
(c) abc (d) $3a^2 b^2 c^2$

66. If A is a matrix of order $p \times q$ and B is a matrix of order $s \times t$, under which one of the following conditions does AB exist ?
(a) $p = t$ (b) $p = s$
(c) $q = t$ (d) $q = s$

67. If A is a square matrix such that $A - A^T = 0$, then which one of the following is correct ?
(a) A must be a null matrix
(b) A must be a unit matrix
(c) A must be a scalar matrix
(d) None of the above

68. What is the largest value of a third order determinant whose elements are 0 or 1 ?
(a) 0   (b) 1
(c) 2   (d) 3

69. If a, b, c are in geometric progression and $a, 2b, 3c$ are in arithmetic progression, then what is the common ratio r such that $0 < r < 1$?

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

70. If a set X contains n ($n > 5$) elements, then what is the number of subsets of X containing less than 5 elements?
(a) $C(n, 4)$ (b) $C(n, 5)$
(c) $\sum\limits_{r=0}^{5} C(n, r)$ (d) $\sum\limits_{r=0}^{4} C(n, r)$

71. Let z be a non-zero complex number. Then, what is z^{-1} (multiplicative inverse of z) equal to ?

(a) $\dfrac{\bar{z}}{|z|^2}$ (b) $\dfrac{z}{|z|^2}$

(c) $\dfrac{\bar{z}}{|z|}$ (d) $\dfrac{|z|}{\bar{z}}$

72. For an AP with first term u and common difference v, the pth term is $15\, uv$ more than the qth term. Which one of the following is correct ?
(a) $p = q + 15\, v$ (b) $p = q + 15\, u$
(c) $p = q + 14\, v$ (d) $p = q + 14\, u$

73. If a, b and c are three positive numbers in an arithmetic progression, then :
(a) $ac > b^2$ (b) $b^2 > a + c$
(c) $ab + bc \leq 2\, ac$ (d) $ab + bc \geq 2\, ac$

74. If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$, then what is $f\left(\dfrac{2x}{1+x^2}\right)$ equal to ?

(a) $(f(x))^2$ (b) 1

(c) $2f(x)$ (d) $f\left(\dfrac{1-x}{1+x}\right)$

75. What is the inverse of $A = \begin{bmatrix} 1+i & 1+i \\ -1+i & 1-i \end{bmatrix}$?

(a) $\dfrac{1}{4}\begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$ (b) $\dfrac{1}{4}\begin{bmatrix} 1+i & -1+i \\ 1+i & -1-i \end{bmatrix}$

(c) $\dfrac{1}{4}\begin{bmatrix} 1+i & 1-i \\ -1-i & 1+i \end{bmatrix}$ (d) $\dfrac{1}{4}\begin{bmatrix} 1+i & 1-i \\ -1-i & -1+i \end{bmatrix}$

76. In respect of the equation

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ c-5 \end{bmatrix};$$

correctly match List I with List II and select the correct answer using the code given below the lists :

List I (Value of c)		List II (Nature of the Equation)
A.	5	1. The equation has no solution
B.	10	2. The equation has a unique solution
C.	15	3. The equation has an infinite set of solutions
		4. The equation has two infinite sets of independent solutions

Code :

	A	B	C
(a)	4	2	3
(b)	1	1	3
(c)	2	2	4
(d)	4	1	3

77. If $A^{-1} = \begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}$, what is det (A) ?

(a) 2   (b) -2

(c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

78. From the matrix equation $AB = AC$, which one of the following can be concluded ?

(a) $B = C$ for any matrix A

(b) $B = C$, if A is singular

(c) $B = C$, if A is non-singular

(d) $A = B = C$ for any matrix A

79. What is the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ if $a^3 + b^3 + c^3 = 0$?

(a) 0   (b) 1

(c) $3abc$ (d) $-3abc$

80. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a 2×2 matrix and $f(x) = x^2 - x + 2$ is a polynomial, then what is $f(A)$?

(a) $\begin{bmatrix} 1 & 7 \\ 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 6 \\ 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 6 \\ 0 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 6 \\ 0 & 7 \end{bmatrix}$

81. If A is a non-null row matrix with 5 columns and B is a non-null column matrix with 5 rows, how many rows are there in $A \times B$?

(a) 1   (b) 5   (c) 10   (d) 25

82. Given that $p = \tan \alpha + \tan \beta$, and $q = \cot \alpha + \cot \beta$; then what is $\left(\dfrac{1}{p} - \dfrac{1}{q}\right)$ equal to ?

(a) $\cot(\alpha - \beta)$ (b) $\tan(\alpha - \beta)$

(c) $\tan(\alpha + \beta)$ (d) $\cot(\alpha + \beta)$

83. If two circles A, B of equal radii pass through the centres of each other, then what is the ratio of the length of the smaller arc to the circumference of the circle A cut off by the circle B ?

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{1}{3}$ (d) $\dfrac{2}{3}$

84. A is a certain positive acute angle which satisfies the following equation :

Number of degrees in A + Number of radians in $A = (180 + \pi)/3$

What is the angle A ?

(a) 20°   (b) 40°   (c) 60°   (d) 80°

85. The angles A, B, C of a triangle are in the ratio $2 : 5 : 5$. What is the value of $\tan B \tan C$?

(a) $4 + \sqrt{3}$ (b) $4 + 2\sqrt{3}$

(c) $7 + 4\sqrt{3}$ (d) $3 + 3\sqrt{3}$

86. If $\sin^3 \theta + \cos^3 \theta = 0$, then what is the value of θ ?

(a) $-\pi/4$ (b) 0

(c) $\pi/4$ (d) $\pi/3$

87. If $0 < x < y < \pi$, then which one of the following is correct?

(a) $x - \cos x > y - \cos y$ (b) $x - \cos x < y - \cos y$

(c) $x + \cos x > y + \cos y$ (d) $x + \cos x < y + \cos y$

88. What is the $(m-1)^{\text{th}}$ root of $\left[(a^m)^m - \left(\dfrac{1}{m}\right)\right]^{\frac{1}{m+1}}$?

(a) $a^{m + (1/m)}$ (b) $a^{m - (1/m)}$

(c) a (d) 1

89. Let $ABCD$ be a parallelogram whose diagonals intersect at P and let O be the origin, then what is $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equal to ?

(a) $\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$

(c) $3\overrightarrow{OP}$ (d) $4\overrightarrow{OP}$

90. If $\overrightarrow{r_1}, \overrightarrow{r_2}, \overrightarrow{r_3}$ are the position vectors of three collinear points and scalars m and n exist such that $\overrightarrow{r_3} = m \overrightarrow{r_1} + n \overrightarrow{r_2}$, then what is the value of $(m + n)$?

(a) 0   (b) 1   (c) -1 (d) 2

91. Let α be the angle which the vector $\overrightarrow{V} = 2\hat{i} - \hat{j} + 2\hat{k}$ makes with the z-axis. Then, what is the value of $\sin \alpha$?

(a) $2/3$ (b) $1/3$

(c) $\sqrt{5}/3$ (d) $\sqrt{5}/9$

92. If $\overrightarrow{m}, \overrightarrow{n}, \overrightarrow{r}$ are three vectors, θ is the angle between the vectors $\overrightarrow{m}$ and $\overrightarrow{n}$, what is $mnr \cos \theta$ equal to ?

(a) $(\overrightarrow{m} \cdot \overrightarrow{n})(\overrightarrow{r}(\overrightarrow{r}/r))$ (b) $(\overrightarrow{m} \cdot \overrightarrow{n})(\overrightarrow{r} \cdot \overrightarrow{r})$

(c) $(\overrightarrow{m} \cdot \overrightarrow{r})(\overrightarrow{n} \cdot (\overrightarrow{n}/n))$ (d) $(\overrightarrow{m} \cdot \overrightarrow{n})\overrightarrow{r}$

93. If the vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then what is the locus of the point (x, y)?

(a) A circle   (b) An ellipse

(c) A parabola   (d) A hyperbola

94. If the components of $\overrightarrow{b}$ along and perpendicular to $\overrightarrow{a}$ are $\lambda\overrightarrow{a}$ and $\overrightarrow{b} - \lambda\overrightarrow{a}$ respectively, what is λ equal to ?

(a) $\dfrac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|}$ (b) $\dfrac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{b}|}$

(c) $\dfrac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|^2}$ (d) $\dfrac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{b}|^2}$

95. A force $m\hat{i} - 3\hat{j} + \hat{k}$ acts on a point and so the point moves from $(20, 3m, 0)$ to $(0, 0, 7)$. If the work done by the force is -48 unit, what is the value of m ?
    (a) 5   (b) 3
    (c) 2   (d) 1

96. If we join the mid points of the upper horizontal sides of each rectangle of a histogram by straight lines, what is the figure so obtained known as ?
    (a) Frequency curve   (b) Frequency polygon
    (c) Ogive (>)   (d) Ogive (<)

97. A firm employing 30 workers and paying on an average Rs 500 is combined with another firm employing 20 workers paying on an average Rs 600. What is the average pay of the workers of the combined firm ?
    (a) Rs 540   (b) Rs 550
    (c) Rs 560   (d) Rs 580

98. The definition of Mode fails if :
    (a) the maximum frequency is repeated
    (b) the maximum frequency is not repeated
    (c) the maximum frequency occurs in the middle
    (d) the curve drawn with the help of given data is symmetrical

99. The middle point of $A(1, 2)$ and $B(x, y)$ is $C(2, 4)$. If BD is perpendicular to AB such that $CD = 3$ unit, then what is the length BD ?
    (a) $2\sqrt{2}$ unit   (b) 2 unit
    (c) 3 unit   (d) $3\sqrt{2}$ unit

100. If the points $A(1, 2)$, $B(2, 4)$ and $C(3, a)$ are collinear, what is the length BC ?
    (a) $\sqrt{2}$ unit   (b) $\sqrt{3}$ unit
    (c) $\sqrt{5}$ unit   (d) 5 unit

101. The bisector of the acute angle between the straight lines $3x - 4y - 3 = 0$ and $12x + 5y + 6 = 0$ passes through which one of the following points ?
    (a) $(5, 3)$ (b) $(-3, 6)$
    (c) $(2, 7)$ (d) $(-1, 4)$

102. If the extremities of a diameter of a circle are $(0, 0)$ and $(a^3, 1/a^3)$, then the circle passes through which one of the following points ?
    (a) $(a^2, 1/a^2)$ (b) $(a, 1/a)$
    (c) $(a, -a)$ (d) $(1/a, a)$

103. What is the length of the intercept made on the x-axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$?
    (a) $\dfrac{\sqrt{(g^2 - c)}}{2}$ (b) $\dfrac{\sqrt{(g^2 - 4c)}}{2}$
    (c) $2\sqrt{(g^2 - 4c)}$ (d) $2\sqrt{(g^2 - c)}$

104. Which one of the following points lies outside the ellipse $(x^2/a^2) + (y^2/b^2) = 1$?
    (a) $(a, 0)$ (b) $(0, b)$
    (c) $(-a, 0)$ (d) (a, b)

105. What is the locus of the point of intersection of the straight lines $(x/a) + (y/b) = m$ and $(x/a) - (y/b) = 1/m$?
    (a) Circle   (b) Parabola
    (c) Ellipse   (d) Hyperbola

106. If the direction ratios of the normal to a plane are $< l, m, n >$ and the length of the normal is p, then what is the sum of intercepts cut-off by the plane from the coordinate axes ?
    (a) $p\left(\dfrac{1}{l} + \dfrac{1}{m} + \dfrac{1}{n}\right)$
    (b) $p\sqrt{(l^2 + m^2 + n^2)}$
    (c) $p\sqrt{(l^2 + m^2 + n^2)}\left(\dfrac{1}{l} + \dfrac{1}{m} + \dfrac{1}{n}\right)$
    (d) $\dfrac{p}{\sqrt{(l^2 + m^2 + n^2)}}\left(\dfrac{1}{l} + \dfrac{1}{m} + \dfrac{1}{n}\right)$

107. How many arbitrary constants are there in the equation of a plane ?
    (a) 2   (b) 3
    (c) 4   (d) Any finite number

108. If P, Q are $(2, 5, -7)$, $(-3, 2, 1)$ respectively, then what are the direction ratios of the line PQ ?
    (a) $< 10, 6, -16 >$ (b) $< 5, 3, 8 >$
    (c) $< -5, -3, -8 >$ (d) None of these

109. If O, P are the points $(0, 0, 0)$, $(2, 3, -1)$ respectively, then what is the equation to the plane through P at right angles to OP ?
    (a) $2x + 3y + z = 16$ (b) $2x + 3y - z = 14$
    (c) $2x + 3y + z = 14$ (d) $2x + 3y - z = 0$

110. The four points $(0, 4, 1)$, $(2, 3, -1)$, $(4, 5, 0)$, $(2, 6, 2)$ are the vertices of which one of the following figures ?
    (a) Rhombus   (b) Rectangle
    (c) Square   (d) Parallelogram

111. If the letters of the word BAZAR are arranged in dictionary order, then what is the 50th word ?
    (a) ZAABR   (b) ZBAAR
    (c) ZBRAA   (d) ZAARB

112. Let $a, b \in \{1, 2, 3\}$. What is the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots ?
    (a) 1   (b) 2
    (c) 5   (d) 3

113. If $px^2 + qx + r = p(x - \alpha)(x - \beta)$, and $p^3 + pq + r = 0; p, q$ and r being real numbers, then which of the following is not possible ?
    (a) $\alpha = \beta = p$ (b) $\alpha \ne \beta = p$
    (c) $\alpha = \beta \ne p$ (d) $\beta \ne \alpha = p$

114. If $|x| < \dfrac{1}{2}$, what is the value of
    $$1 + n\left[\dfrac{x}{1-x}\right] + \left[\dfrac{n(n+1)}{2!}\right]\left[\dfrac{x}{1-x}\right]^2 + \cdots\cdots\cdots \infty ?$$
    (a) $\left[\dfrac{1-x}{1-2x}\right]^n$ (b) $(1-x)^n$
    (c) $\left[\dfrac{1-2x}{1-x}\right]^n$ (d) $\left(\dfrac{1}{1-x}\right)^n$

115. What are the last two digits of the number 9^{200} ?
    (a) 19   (b) 21   (c) 41   (d) 01

116. For any positive integer n, if $4^n - 3n$ is divided by 9, then what is the remainder ?
    (a) 8   (b) 6   (c) 4   (d) 1

117. In how many ways can 7 persons stand in the form of a ring ?
    (a) $P(7, 2)$ (b) $7!$
    (c) $6!$ (d) $\dfrac{7!}{2}$

118. The sum of the first $(2p+1)$ terms of an AP is $\{(p+1) \cdot (2p+1)\}$. Which one of the following inferences can be drawn ?
  (a) The $(p+1)^{\text{th}}$ term of the AP is $(2p+1)$
  (b) The $(2p+1)^{\text{th}}$ term of the AP is $(2p+1)$
  (c) The $(2p+1)^{\text{th}}$ term of the AP is $(p+1)$
  (d) The $(p+1)^{\text{th}}$ term of the AP is $(p+1)$

119. Which one of the following is an infinite set ?
  (a) The set of human beings on the earth
  (b) The set of water drops in a glass of water
  (c) The set of trees in a forest
  (d) The set of all primes

120. What is the value of $0.\bar{2}+0.\overline{23}$?
  (a) $0.\overline{43}$
  (b) $0.\overline{45}$
  (c) $0.\overline{223}$
  (d) $0.\overline{223}$

Answer: Mathematics II

1	(a)	51	(a)	101	(c)
2	(a)	52	(d)	102	(d)
3	(a)	53	(d)	103	(d)
4	(a)	54	(a)	104	(d)
5	(c)	55	(c)	105	(d)
6	(a)	56	(d)	106	(a)
7	(a)	57	(d)	107	(c)
8	(c)	58	(a)	108	(c)
9	(c)	59	(d)	109	(b)
10	(c)	60	(c)	110	(c)
11	(c)	61	(c)	111	(d)
12	(c)	62	(d)	112	(d)
13	(d)	63	(c)	113	(a)
14	(b)	64	(b)	114	(a)
15	(c)	65	(b)	115	(d)
16	(b)	66	(d)	116	(d)
17	(c)	67	(d)	117	(c)
18	(c)	68	(c)	118	(d)
19	(c)	69	(a)	119	(d)
20	(d)	70	(d)	120	(b)
21	(d)	71	(a)		
22	(d)	72	(b)		
23	(b)	73	(d)		
24	(b)	74	(c)		
25	(c)	75	(a)		
26	(a)	76	(b)		
27	(a)	77	(d)		
28	(c)	78	(c)		
29	(b)	79	(c)		
30	(b)	80	(b)		
31	(a)	81	(a)		
32	(b)	82	(d)		
33	(a)	83	(c)		
34	(a)	84	(c)		
35	(b)	85	(c)		
36	(b)	86	(a)		
37	(c)	87	(b)		
38	(c)	88	(c)		

39	(a)		89	(d)
40	(c)		90	(b)
41	(c)		91	(c)
42	(c)		92	(d)
43	(c)		93	(a)
44	(b)		94	(c)
45	(a)		95	(a)
46	(c)		96	(b)
47	(d)		97	(a)
48	(c)		98	(d)
49	(c)		99	(b)
50	(a)		100	(c)