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UPSC NDA Exam 2006-I Mathematics Solved Paper

Mathematics

1. Universal set,

$$U=\left\{x\middle|x^5-6x^4+11x^3-6x^2=0\right\}$$

$$A=\left\{x\middle|x^2-5x+6=0\right\}$$

$$B=\left\{x\middle|x^2-3x+2=0\right\}$$

What is $(A\cap B)'$ equal to ?

(a) {1, 3} (b) {1, 2, 3}

(c) {0, 1, 3} (d) {0, 1, 2, 3}

2. If the sum of first 10 terms of an arithmetic progression with first term p and common difference q, is 4 times the sum of the first 5 terms, then what is the ratio $p:q$?

(a) 1 : 2 (b) 1 : 4

(c) 2 : 1 (d) 4 : 1

3. If the roots of the equation $4\beta^2+\lambda\beta-2=0$ are of the form $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$, then what is the value of λ ?

(a) $2k$ (b) 7

(c) 2 (d) $k+1$

4. Suppose that A denotes the collection of all complex numbers whose square is a negative real number. Which one of the following statements is correct ?

(a) $A\subseteq R$

(b) $A\supseteq R$

(c) $A=\left\{x+iy\middle|x^2\in R\middle|x,\ y\subseteq R\right\}$

(d) $A=\{iy|y\in R\}$

5. A relation R is defined on the set Z of integers as follows $mRn \Leftrightarrow m+n$ is odd.

Which of the following statements is/are true for R ?

1. R is reflexive

2. R is symmetric

3. R is transitive

Select the correct answer using the code given below :

(a) 2 only (b) 2 and 3

(c) 1 and 2 (d) 1 and 3

6. One of the roots of a quadratic equation with real coefficients is $\frac{1}{(2-3i)}$. Which of the following implications is/are true ?

1. The second root of the equation will be $\frac{1}{(3-2i)}$.

2. The equation has no real root.

3. The equation is $13x^2-4x+1=0$.

Which of the above is/are correct ?

(a) 1 and 2 only

(b) 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

7. Given $4a-2b+c=0$, where $a, b, c\in R$, which of the following statements is/are not true in general ?

1. $(x+2)$ will always be a factor of the expression ax^2+bx+c.

2. $(x-2)$ will always be a factor of the expression ax^2+bx+c.

3. There will be a factor of the expression ax^2+bx+c different from $(x+2)$.

Select the correct answer using the code given below

(a) 1 and 2 only (b) 1, 2 and 3

(c) 2 only (d) 1 only

8. If the sum of the squares of the roots of $x^2-(p-2)x-(p+1)=0$ $(p\in R)$ is 5, then what is the value of p ?

(a) 0 (b) -1

(c) 1 (d) $\frac{3}{2}$

9. Let A and B be two non-empty subsets of a set X. If $(A-B)\cup(B-A)=A\cup B$, then which one of the following is correct ?

(a) $A\subset B$ (b) $A\subset(X-B)$

(c) $A=B$ (d) $B\subset A$

10. Let $A=\{(n, 2n): n\in N\}$ and $B=\{(2n, 3n): n\in N\}$, What is $A\cap B$ equal to ?

(a) $\{(n, 6n): n\in N\}$

(b) $\{(2n, 6n): n\in N\}$

(c) $\{(n, 3n): n\in N\}$

(d) ϕ

11. Which one of the following operations on sets is not correct where B' denotes the complement of B ?

(a) $(B'-A')\cup(A'-B')=(A\cup B)-(A\cap B)$

(b) $(A-B)\cup(B-A)=(A'\cup B')-(A'\cap B')$

(c) $(B'-A')\cap(A'-B')=(B-A)\cap(A-B)$

(d) $(B'-A')\cap(A'-B')=(B-A')\cup(A'-B)$

12. Which one of the following sets has all elements as odd positive integers ?

(a) $S=\left\{x\in R\middle|x^3-8x^2+19x-12=0\right\}$

(b) $S=\left\{x\in R\middle|x^3-9x^2+23x-15=0\right\}$

(c) $S=\left\{x\in R\middle|x^3-7x^2+14x-8=0\right\}$

(d) $S=\left\{x\in R\middle|x^3-12x^2+44x-48=0\right\}$

13. Which of the following statements is not correct for the relation R defined by aRb if and only if b lives within one kilometre from a ?

(a) R is reflexive

(b) R is symmetric

(c) R is not anti-symmetric

(d) None of the above

14. Let X be any non-empty set containing n elements. Then what is the number of relations on X ?

(a) 2^{n^2} (b) 2^n
(c) 2^{2n} (d) n^2

15. What is the sum of the first 50 terms of the series $(1\times 3)+(3\times 5)+(5\times 7)+\ldots.$?

(a) 1,71,650 (b) 26,600
(c) 26,650 (d) 26,900

16. What is the number of real solutions of $|x^2-x-6|=x+2$?

(a) 4 (b) 3 (c) 2 (d) 1

17. What is the region that represents $A\cap B$ if $A=\{(x,y)|x+y\le 4\}$ and $B=\{(x,y)|x+y\le 0\}$?

(a) $\{(x,y)|x+y\le 2\}$ (b) $\{(x,y)|2x+y\le 4\}$
(c) $\{(x,y)|x+y\le 0\}$ (d) $\{(x,y)|x+y\le 4\}$

18. How many 3-digit numbers, each less than 600, can be formed from {1, 2, 3, 4, 7, 9} if repetition of digits is allowed ?

(a) 216 (b) 180
(c) 144 (d) 120

19. If the roots of $x^2-2mx+m^2-1=0$ lie between -2 and 4, then which one of the following is correct ?

(a) $-1\le m\le 3$ (b) $-3\le m\le 3$
(c) $-3\le m\le 5$ (d) $-1\le m\le 5$

20. If $(\log_3 x)^2+\log_3 x<2$, then which one of the following is correct ?

(a) $0<x<\frac{1}{9}$ (b) $\frac{1}{9}<x<3$
(c) $3<x<\infty$ (d) $\frac{1}{9}\le x\le 3$

21. In a group of 500 students, there are 475 students who can speak Hindi and 200 can speak Bengali. What is the number of students who can speak Hindi only ?

(a) 275 (b) 300
(c) 325 (d) 350

22. Let X and Y be two non-empty sets and let R_1 and R_2 be two relations from X into Y. Then, which one of the following is correct ?

(a) $(R_1\cap R_2)^{-1}\subset R_1^{-1}\cap R_2^{-1}$
(b) $(R_1\cap R_2)^{-1}\supset R_1^{-1}\cap R_2^{-1}$
(c) $(R_1\cap R_2)^{-1}=R_1^{-1}\cap R_2^{-1}$
(d) $(R_1\cap R_2)^{-1}=R_i^{-1}\cup R_2^{-1}$

23. What is the value of
$$\frac{(1001)_2{}^{(11)2}-(101)_2{}^{(11)2}}{(1001)_2{}^{(10)2}+(1001)_2{}^{(01)2}\,(101)_2{}^{(01)2}+(101)_2{}^{(10)2}}\ ?$$

(a) $(1001)_2$ (b) $(101)_2$
(c) $(110)_2$ (d) $(100)_2$

24. Let $x>y$ be two real numbers and $z\in R$, $z\neq 0$. Consider the following

1. $x+z>y+z$ and $xz>yz$
2. $x+z>y+z$ and $x-z>y-z$
3. $xz>yz$ and $\frac{x}{z}>\frac{y}{z}$
4. $x-z>y-z$ and $\frac{x}{z}>\frac{y}{z}$

Which of the above is/are correct ?

(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2, 3 and 4

25. If z_1, z_2 are any two complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, which one of the following is correct ?

(a) $z_1=\alpha z_2$ where $\alpha\in R$ (b) $z_1\ge 0$ or $z_2\ge 0$
(c) $z_1=\alpha z_2$ where $\alpha>0$ (d) $|z_1|=|z_2|$

26. If $x=1+\frac{y}{2}+\left(\frac{y}{2}\right)^2+\left(\frac{y}{2}\right)^3+\ldots.$ where $|y|<2$, what is y equal to ?

(a) $\frac{x-1}{x}$ (b) $\frac{x-1}{2x}$
(c) $\frac{2x-2}{x}$ (d) $\frac{2x+1}{2x}$

27. There are four chairs with two chairs in each row. In how many ways can four persons be seated on the chairs, so that no chair remains unoccupied ?

(a) 6 (b) 12
(c) 24 (d) 48

28. In how many ways can the letters of the word CORPORATION be arranged so that vowels always occupy even places ?

(a) 120 (b) 2700
(c) 720 (d) 7200

29. If all permutations of the letters of the word 'LAGAN' are arranged as in dictionary, then what is the rank of 'NAAGL' ?

(a) 48th word (b) 49th word
(c) 50th word (d) 51st word

30. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$?

(a) -272 (b) -540
(c) -870 (d) -918

31. $A_{(\alpha)}=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$,

$A_{(\beta)}=\begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix}$

Which one of the following is correct ?

(a) $A_{(-\alpha)}\,A_{(-\beta)}=A_{(\alpha+\beta)}$
(b) $A_{(-\alpha)}\,A_{(\beta)}=A_{(\beta-\alpha)}$
(c) $A_{(\alpha)}+A_{(-\beta)}=A_{(-(\beta-\alpha))}$
(d) $A_{(\alpha)}+A_{(\beta)}=A_{(\alpha+\beta)}$

32. If
$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x\\ \sin^2 x & 1+\cos^2 x & 4\sin 2x\\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$$

What is the maximum value of $f(x)$?

(a) 2 (b) 4
(c) 6 (d) 8

33. If the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular, then what is one of the values of θ ?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) 0

34. For what values of k, does the system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ have a unique solution ?

(a) $k = 0$ (b) $-1 < k < 1$

(c) $-2 < k < 2$ (d) $k \neq 0$

35. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$

If $AB = BA$, then what is the value of x ?

(a) -1 (b) 0

(c) 1 (d) Any real number

36. If a matrix B is obtained from a square matrix A by interchanging any two of its rows, then what is $|A + B|$ equal to

(a) $2|A|$ (b) $2|B|$

(c) 0 (d) $|A| - |B|$

37. Let $A = (a_{ij})_{n \times n}$ and adj $A = (\alpha_{ij})$

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 2 & 3 & -1 \end{bmatrix}$, What is the value of α_{23} ?

(a) 1 (b) -1

(c) 8 (d) -8

38. Let A and B be two invertible square matrices each of order n. What is $adj\,(AB)$ equal to ?

(a) (adj A) (adj B) (b) (adj A) + (adj B)

(c) (adj A) − (adj B) (d) (adj B) (adj A)

39. M is a matrix with real entries given by

$$M = \begin{bmatrix} 4 & k & 0 \\ 6 & 3 & 0 \\ 2 & t & k \end{bmatrix}$$

Which of the following conditions guarantee the invertibility of M ?

1. $k \neq 2$
2. $k \neq 0$
3. $t \neq 0$
4. $t \neq 1$

Select the correct answer using the code given below

(a) 1 and 2 (b) 2 and 3

(c) 1 and 4 (d) 3 and 4

40. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ be a square matrix of order 3. Then, for any positive integer n, what is A^n equal to ?

(a) A (b) $3^n A$

(c) $(3^{n-1})A$ (d) $3A$

41. Let A and B be two matrices such that AB is defined. If $AB = 0$, then which one of the following can be definitely concluded ?

(a) $A = 0$ or $B = 0$

(b) $A = 0$ and $B = 0$

(c) A and B are non-zero square matrices

(d) A and B cannot both be non-singular

42. The difference of two angles is 1°; the circular measure of their sum is 1. What is the smaller angle in circular measure ?

(a) $\left[\frac{180°}{\pi} - 1\right]$ (b) $\left[1 - \frac{\pi}{180°}\right]$

(c) $\frac{1}{2}\left[1 - \frac{\pi}{180°}\right]$ (d) $\frac{1}{2}\left[\frac{180°}{\pi} - 1\right]$

43. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is $\frac{1}{2}$. What is the height of the pole ?

(a) 110 m (b) 200 m

(c) 120 m (d) 150 m

44. In a triangle ABC, $a = 2b$ and $\angle A = 3\angle B$. Which one of the following is correct ?

(a) The triangle is isosceles

(b) The triangle is equilateral

(c) The triangle is right-angled

(d) Such triangle does not exist

45. A positive acute angle is divided into two parts whose tangents are $\frac{1}{8}$ and $\frac{7}{9}$. What is the value of this angle ?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

46. If an angle B is complement of an angle A, what are the greatest and least values of cos A cos B respectively ?

(a) $0, -\frac{1}{2}$ (b) $\frac{1}{2}, -1$

(c) 1, 0 (d) $\frac{1}{2}, -\frac{1}{2}$

47. Three expressions are given below

$Q_1 = \sin(A + B) + \sin(B + C) + \sin(C + A)$

$Q_2 = \cos(A - B) + \cos(B - C) + \cos(C - A)$

$Q_3 = \sin A(\cos B + \cos C) + \sin B(\cos C + \cos A) + \sin C(\cos A + \cos B)$

Which one of the following is correct ?

(a) $Q_1 = Q_2$

(b) $Q_2 = Q_3$

(c) $Q_1 = Q_3$

(d) All the expressions are different

48. For what values of x is the equation $2\sin\theta = x + \frac{1}{x}$ valid ?

(a) $x = \pm 1$ (b) All real values of x

(c) $-1 < x < 1$ (d) $x > 1$ and $x < -1$

49. If $\sin(\pi \cos x) = \cos(\pi \sin x)$, then what is one of the values of $\sin 2x$?

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$

(c) $-\frac{3}{4}$ (d) -1

50. What is the value of

$\tan(\tan^{-1} x + \tan^{-1} y + \tan^{-1} z) -$

$\cot(\cot^{-1} x + \cot^{-1} y + \cot^{-1} z)$?

(a) 0 (b) $2(x + y + z)$

(c) $\frac{3\pi}{2}$ (d) $\frac{3\pi}{2} + x + y + z$

51. In a triangle ABC, if $\cos A = \cos B \cos C$, what is the value of $\tan A - \tan B - \tan C$?

(a) 0

(b) -1

(c) $1 + \tan A \tan B \tan C$

(d) $\tan A \tan B \tan C - 1$

52. What is the value of x that satisfies the equation $\cos^{-1} x = 2\sin^{-1} x$?

(a) $\frac{1}{2}$ (b) -1

(c) 1 (d) $-\frac{1}{2}$

53. OAB is a given triangle such that $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$. Also C is a point on $\vec{AB}$ such that $\vec{AB} = 2\vec{BC}$. What is $\vec{AC}$ equal to ?

(a) $\frac{1}{2}(\vec{b} - \vec{a})$ (b) $\frac{1}{2}(\vec{b} + \vec{a})$

(c) $\frac{3}{2}(\vec{a} - \vec{b})$ (d) $\frac{3}{2}(\vec{b} - \vec{a})$

54. Let $\vec{a}, \vec{b}, \vec{c}$ be non-coplanar vectors and

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]},$$

What is the value of

$(\vec{a} - \vec{b} - \vec{c}) \cdot \vec{p} + (\vec{b} - \vec{c} - \vec{a}) \cdot \vec{q} + (\vec{c} - \vec{a} - \vec{b}) \cdot \vec{r}$?

(a) 0 (b) -3

(c) 3 (d) -9

55. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of corners A, B, C of a parallelogram $ABCD$, then what is the position vector of the corner D ?

(a) $\vec{a} + \vec{b} + \vec{c}$ (b) $\vec{a} + \vec{b} - \vec{c}$

(c) $\vec{a} - \vec{b} + \vec{c}$ (d) $-\vec{a} + \vec{b} + \vec{c}$

56. In a ΔABC, angle B is obtuse and D, E, F are the middle points of sides BC, CA, AB respectively. Which one of the following vectors has the greatest magnitude ?

(a) $\vec{BC}$ (b) $\vec{CA}$

(c) $\vec{AB}$ (d) $\vec{AD}$

57. If $\vec{p} \neq \vec{0}$ and the conditions $\vec{p} \cdot \vec{q} = \vec{p} \cdot \vec{r}$ and $\vec{p} \times \vec{q} = \vec{p} \times \vec{r}$ hold simultaneously, then which one of the following is correct ?

(a) $\vec{q} \neq \vec{r}$ (b) $\vec{q} = -\vec{r}$

(c) $\vec{q} \cdot \vec{r} = 0$ (d) $\vec{q} = \vec{r}$

58. If two unit vectors $\vec{p}$ and $\vec{q}$ make an angle $\frac{\pi}{3}$ with each other, what is the magnitude of $\vec{p} - \frac{1}{2}\vec{q}$?

(a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$

59. The median AD of a triangle ABC is bisected at F, and BF is produced to meet the side AC in P. If $AP = \lambda AC$, then what is the value of λ ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

60. What are the values of x for which the two vectors $(x^2 - 1)\hat{i} + (x + 2)\hat{j} + x^2\hat{k}$ and $2\hat{i} - x\hat{j} + 3\hat{k}$ are orthogonal ?

(a) No real value of x (b) $x = \frac{1}{2}$ and $x = -1$

(c) $x = -\frac{1}{2}$ and $x = 1$ (d) $x = -1$ and $x = 2$

61. What is the moment about the point $\hat{i} + 2\hat{j} + 3\hat{k}$, of a force represented by $\hat{i} + \hat{j} + \hat{k}$, acting through the point $-2\hat{i} + 3\hat{j} + \hat{k}$?

(a) $2\hat{i} + \hat{j} + 2\hat{k}$ (b) $\hat{i} - \hat{j} + 3\hat{k}$

(c) $3\hat{i} + 2\hat{j} - \hat{k}$ (d) $3\hat{i} + \hat{j} - 4\hat{k}$

62. A particle is acted upon by following forces

(i) $2\hat{i} + 3\hat{j} + 5\hat{k}$

(ii) $-5\hat{i} + 4\hat{j} - 3\hat{k}$ and

(iii) $3\hat{i} - 7\hat{k}$

In which plane does it move ?

(a) x-y plane

(b) y-z plane

(c) z-x plane

(d) Any arbitrary plane

63. What is the vector whose magnitude is 3, and is perpendicular to $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$?

(a) $3(\hat{i} + \hat{j} - \hat{k})$ (b) $\sqrt{3}(\hat{i} - \hat{j} + \hat{k})$

(c) $\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ (d) $3(\hat{i} - \hat{j} + \hat{k})$

64. An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with one of the vertices at $(a, 0)$. What is the equation of the side opposite to this vertex ?

(a) $2x - a = 0$ (b) $x + a = 0$

(c) $2x + a = 0$ (d) $3x - 2a = 0$

65. Consider the points $(a-1, a, a+1)$, $(a, a+1, a-1)$ and $(a+1, a-1, a)$.
1. These points always form the vertices of an equilateral triangle for any real value of a.
2. The area of the triangle formed by these points is independent of a.
Which of the statement(s) given above is/are correct ?
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

66. What are coordinates of the point equidistant from the points $(a, 0, 0)$, $(0, a, 0)$, $(0, 0, a)$ and $(0, 0, 0)$?
(a) $\left(\frac{a}{3}, \frac{a}{3}, \frac{a}{3}\right)$ (b) $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$
(c) (a, a, a) (d) $(2a, 2a, 2a)$

67. If the latusrectum of an ellipse is equal to one half its minor axis, what is the eccentricity of the ellipse ?
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{3}{4}$ (d) $\frac{\sqrt{15}}{4}$

68. A line makes 45° with positive x-axis and makes equal angles with positive y, z axes, respectively. What is the sum of the three angles which the line makes with positive x, y and z axes ?
(a) 180° (b) 165°
(c) 150° (d) 135°

69. $P(2, 2)$ is a point on the parabola $y^2 = 2x$ and A is its vertex. Q is another point on the parabola such that PQ is perpendicular to AP. What is the length of PQ ?
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $4\sqrt{2}$ (d) $6\sqrt{2}$

70. What is the radius of the circle passing through the points $(0, 0)$, $(a, 0)$ and $(0, b)$?
(a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
(c) $\frac{1}{2}\sqrt{a^2 + b^2}$ (d) $2\sqrt{a^2 + b^2}$

71. The focal distance of a point on the parabola $y^2 = 12x$ is 4. What is the abscissa of the point ?
(a) 1 (b) –1
(c) $2\sqrt{3}$ (d) –2

72. What is the angle between the two lines whose direction numbers are
$(\sqrt{3}-1, -\sqrt{3}-1, 4)$ and $(-\sqrt{3}-1, \sqrt{3}-1, 4)$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

73. If $(2, 0)$ is the vertex and the y-axis is the directrix of a parabola, then where is its focus ?
(a) $(0, 0)$ (b) $(-2, 0)$
(c) $(4, 0)$ (d) $(-4, 0)$

74. The lines $(p+2q)x + (p-3q)y = p-q$ for different values of p and q pass through the fixed point given by which one of the following ?
(a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (b) $\left(\frac{2}{5}, \frac{2}{5}\right)$
(c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ (d) $\left(\frac{2}{5}, \frac{3}{5}\right)$

75. What is the angle between the two straight lines $y = (2-\sqrt{3})x + 5$ and $y = (2+\sqrt{3})x - 7$?
(a) 60° (b) 45°
(c) 30° (d) 15°

76. If α, β, γ be angles which the vector $\vec{r} = \lambda\hat{i} + 2\hat{j} - \hat{k}$ makes with the coordinate axes, then what is the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$?
(a) 2 (b) 1
(c) $\lambda^2 + 1$ (d) $1 - \lambda^2$

77. Let R be the set of real numbers and let $f : R \to R$ be a function such that $f(x) = \frac{x^2}{1+x^2}$. What is the range of f ?
(a) R (b) $R - \{1\}$
(c) $[0, 1]$ (d) $[0, 1)$

78. If $u = \sin^{-1}(x-y)$, $x = 3t$, $y = 4t^3$, then what is the derivative of u with respect to t ?
(a) $3(1-t^2)$ (b) $3(1-t^2)^{-\frac{1}{2}}$
(c) $5(1-t^2)^{\frac{1}{2}}$ (d) $5(1-t^2)$

79. Let $f(x) = \frac{1}{\sqrt{18-x^2}}$.
What is the value of $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$?
(a) 0 (b) $-\frac{1}{9}$
(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

80. Let $f(x+y) = f(x)\cdot f(y)$ and $f(1) = 2$ for all $x, y \in R$, where $f(x)$ is continuous function. What is $f'(1)$ equal to ?
(a) $2 \ln 2$ (b) $\ln 2$
(c) 1 (d) 0

81. Given $f(x) = \log\left[\frac{(1+x)}{(1-x)}\right]$ and $g(x) = \frac{(3x+x^3)}{1+3x^2}$,
then what is $f[g(x)]$ equal to ?
(a) $-f(x)$ (b) $3[f(x)]$
(c) $[f(x)]^3$ (d) $-3[f(x)]$

82. Under what condition is the tangent to a given curve at a point perpendicular to x-axis ?
(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$
(c) $\frac{dx}{dy} = 0$ (d) $\frac{d^2y}{dx^2} = 1$

83. If $x = \cos t$, $y = \sin t$, then what is $\frac{d^2y}{dx^2}$ equal to ?

(a) y^{-3} (b) y^3

(c) $-y^{-3}$ (d) $-y^3$

84. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then what is $f'(x_0)$ equal to ?

(a) $\phi'(x_0)$ (b) $\phi(x_0)$

(c) $x_0\phi(x_0)$ (d) $2\phi(x_0)$

85. If $y = x + e^x$, then what is $\frac{d^2x}{dy^2}$ equal to ?

(a) e^x (b) $-\frac{e^x}{(1+e^x)^3}$

(c) $-\frac{e^x}{(1+e^x)}$ (d) $-\frac{e^x}{(1+e^x)^2}$

86. What is the derivative of $f(x) = x|x|$?

(a) $|x| + x$ (b) $2x$

(c) $2|x|$ (d) $-2|x|$

87. If $x + y = t - \frac{1}{t}$, $x^2 + y^2 = t^2 + \frac{1}{t^2}$, what is $\frac{dy}{dx}$ equal to ?

(a) $\frac{1}{x}$ (b) $-\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) $-\frac{1}{x^2}$

88. What does the solution of the differential equation $xdy - ydx = 0$ represent ?

(a) Rectangular hyperbola

(b) Straight line passing through (0, 0)

(c) Parabola with vertex at (0, 0)

(d) Circle with centre at (0, 0)

89. If $f(x) = \ln(x - \sqrt{1+x^2})$, then what is $\int f''(x)\,dx$ equal to ?

(a) $\frac{1}{(x-\sqrt{1+x^2})} + c$ (b) $-\frac{1}{\sqrt{1+x^2}} + c$

(c) $-\sqrt{1+x^2} + c$ (d) $\ln(x - \sqrt{1+x^2}) + c$

90. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$ and

$f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then

What is the value of B ?

(a) $\frac{2}{\pi}$ (b) $\frac{4}{\pi}$

(c) 0 (d) 1

91. If m and n are integers, then what is the value of $\int_0^{\pi} \sin mx \sin nx\,dx$, if $m \neq n$?

(a) 0 (b) $\frac{1}{m+n}$

(c) $\frac{1}{m-n}$ (d) mn

92. Which one of the following differential equations represents the system of circles touching y-axis at the origin ?

(a) $\frac{dy}{dx} = x^2 - y^2$ (b) $2xy\frac{dy}{dx} = y^2 - x^2$

(c) $2xy\frac{dy}{dx} = x^2 - y^2$ (d) $\frac{dy}{dx} = y^2 - x^2$

93. What is the solution of the differential equation $\frac{dy}{dx} = \frac{y}{(x + 2y^3)}$?

(a) $y(1 - xy) = cx$ (b) $y^3 - x = cy$

(c) $x(1 - xy) = cy$ (d) $x(1 + xy) = cy$

94. If $y^2 = p(x)$ is a polynomial of degree 3, then what is $2\frac{d}{dx}\left[y^3\frac{d^2y}{dx^2}\right]$ equal to ?

(a) $p'(x)p'''(x)$ (b) $p''(x)p'''(x)$

(c) $p(x)p'''(x)$ (d) A constant

95. What is the degree of the equation

$\left[\frac{d^2y}{dx^2}\right] = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$?

(a) 1 (b) 2

(c) 3 (d) 4

96. The sum of two numbers is 20. What are the numbers if the product of the square of one and the cube of the other is maximum ?

(a) 6 , 14 (b) 15 , 5

(c) 12, 8 (d) 10, 10

97. What is the slope of the normal at the point $(at^2, 2at)$ of the parabola $y^2 = 4ax$?

(a) $\frac{1}{t}$ (b) t

(c) $-t$ (d) $-\frac{1}{t}$

98. What is the area under the curve $y = |x| + |x - 1|$ between $x = 0$ and $x = 1$?

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

99. What is the value of $\lim_{x\to 0}\frac{\sin|x|}{x}$?

(a) 1

(b) -1

(c) ∞

(d) Limit does not exist

100. The production of food grains in Maharashtra is given for the 12 yr from 1992 to 2003. Which one of the following representations is most suitable to depict the data ?

(a) A simple bar diagram

(b) A pie diagram

(c) A component bar diagram with the components arranged in chronological order

(d) A broken line graph

101. In a manufacture of ready-made garments, which average is used to find the most frequent size ?
(a) Arithmetic mean (b) Geometric mean
(c) Mode (d) Harmonic mean

102. If A, B and C are any three arbitrary events, then which one of the following expressions shows that both A and B occur but not C ?
(a) $A \cap \bar{B} \cap \bar{C}$ (b) $A \cap B \cap \bar{C}$
(c) $\bar{A} \cap \bar{B} \cap \bar{C}$ (d) $(A \cup B) \cap \bar{C}$

103. From past experience it is known that an investor will invest in security A with a probability of 0.6, will invest in security B with a probability 0.3 and will invest in both A and B with a probability of 0.2. What is the probability that an investor will invest neither in A nor in B ?
(a) 0.7 (b) 0.28
(c) 0.3 (d) 0.4

104. Five coins whose faces are marked 2, 3 are thrown. What is the probability of obtaining a total of 12 ?
(a) $\frac{1}{16}$ (b) $\frac{3}{16}$
(c) $\frac{5}{16}$ (d) $\frac{7}{16}$

105. Under what condition will the angle between two regression lines become zero ?
(a) $r = 0$ (b) Only when $r = +1$
(c) Only when $r = -1$ (d) $r = \pm 1$

106. What is the arithmetic mean of the series ${}^nC_0, {}^nC_1, \ldots\ldots\ldots {}^nC_n$?
(a) $\frac{2^n}{n}$ (b) $\frac{2^n}{(n+1)}$
(c) $\frac{2^{(n+1)}}{n}$ (d) $\frac{2^{(n+1)}}{(n+1)}$

107. The standard deviation of n observations $x_1, x_2, \ldots\ldots, x_n$ is 6. The standard deviation of another set of n observations $y_1, y_2, \ldots\ldots, y_n$ is 8. What is the standard deviation of n observations $x_1 - y_1, x_2 - y_2, \ldots\ldots, x_n - y_n$?
(a) 10 (b) 7
(c) 14 (d) 2

108. Following is the frequency distribution of life length in hours of 100 electric bulbs

Life length of bulbs (in hrs)	8.5-13.5	13.5-18.5	18.5-23.5	23.5-28.5	28.5-33.5	33.5-38.5
No. of bulbs	7	x	40	y	10	2

If the median of life length is 20 h, then what are the missing frequencies (x, y) ?
(a) (28, 13) (b) (23, 18)
(c) (31, 10) (d) (25, 16)

109. If α, β are real, what is $\left|\frac{\alpha + i\beta}{\beta + i\alpha}\right|$ equal to ?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

110. What is the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?
(a) $-\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $-\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

111. If $z = 1 + i$, then what is the inverse of z^2 ?
(a) $2i$ (b) i
(c) $\frac{i}{2}$ (d) $-\frac{i}{2}$

112. Let $P = \{p_1, p_2, p_3, p_4\}$,
$Q = \{q_1, q_2, q_3, q_4\}$ and
$R = \{r_1, r_2, r_3, r_4\}$.
If $S_{10} = \{(p_i, q_j, r_k) : i + j + k = 10\}$, how many elements does S_{10} have ?
(a) 2 (b) 4
(c) 6 (d) 8

113. Which one of the following is correct ?
(a) $A \cup (B - C) = A \cap (B \cap C')$
(b) $A - (B \cup C) = (A \cap B') \cap C'$
(c) $A - (B \cap C) = (A \cap B') \cap C$
(d) $A \cap (B - C) = (A \cap B) \cap C$

114. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected ?
(a) 110 (b) 55
(c) 22 (d) 11

115. What is the value of $\sqrt{3}$ cosec $20° - \sec 20°$?
(a) 4 (b) 2
(c) 1 (d) 0

116. What is the product of first $2n + 1$ terms of a geometric progression ?
(a) The $(n + 1)$ th power of the nth term of the GP
(b) The $(2n + 1)$ th power of the nth term of the GP
(c) The $(2n + 1)$ th power of the $(n + 1)$ th term of the GP
(d) The nth power of the $(n + 1)$ th term of the GP

Directions : The next 4 (four) items consist of two statements, one labelled as the **'Assertion (A)'** and the other as **'Reason (R)'**. You are to examine these two statements carefully and select the answers to these items using the codes given below

Code :
(A) Both **A** and **R** are individually true and **R** is the correct explanation of **A**
(B) Both **A** and **R** are individually true but **R** is not the correct explanation of **A**
(C) **A** is true but **R** is false
(D) **A** is false but **R** is true

117. **Assertion (A) :** We cannot find out the regression equation of x on y from that of y on x.
Reason (R) : In one equation x is dependent variable and y is independent whereas in other equation y is dependent variable and x is independent.
(a) A (b) B
(c) C (d) D

118. **Assertion (A) :** If $P(A)=\frac{3}{4}$ and $P(B)=\frac{3}{8}$, then

$$P(A \cup B) \geq \frac{3}{4}.$$

Reason (R) : $P(A) \leq P(A \cup B)$ and
$P(B) \leq P(A \cup B)$; hence
$P(A \cup B) \geq \max \{P(A), P(B)\}$

(a) A (b) B (c) C (d) D

119. **Assertion (A) :** If $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$,

then $|\vec{a}| \neq |\vec{b}|$

Reason (R) : Two inequal vectors can never have same magnitude.

(a) A (b) B
(c) C (d) D

120. **Assertion (A) :** $\int_1^e \ln^2 x \, dx = e - 2$

Reason (R) : $I_n = \int_1^e \ln^n x \, dx = e - 2I_{n-1}$

(a) A (b) B
(c) C (d) D

Answer: Mathematics

1	(c)	51	(a)	101	(c)
2	(a)	52	(a)	102	(d)
3	(b)	53	(a)	103	(c)
4	(d)	54	(c)	104	(c)
5	(a)	55	(c)	105	(d)
6	(c)	56	(b)	106	(b)
7	(c)	57	(d)	107	(d)
8	(c)	58	(b)	108	(c)
9	(b)	59	(d)	109	(c)
10	(d)	60	(c)	110	(d)
11	(a)	61	(d)	111	(d)
12	(b)	62	(b)	112	(c)
13	(c)	63	(b)	113	(b)
14	(a)	64	(c)	114	(b)
15	(a)	65	(c)	115	(a)
16	(b)	66	(b)	116	(c)
17	(c)	67	(b)	117	(a)
18	(c)	68	(b)	118	(b)
19	(d)	69	(d)	119	(d)
20	(b)	70	(c)	120	(a)
21	(b)	71	(a)		
22	(d)	72	(c)		
23	(d)	73	(c)		
24	(d)	74	(d)		
25	(a)	75	(a)		
26	(c)	76	(a)		
27	(c)	77	(a)		
28	(d)	78	(b)		
29	(b)	79	(d)		
30	(d)	80	(a)		
31	(a)	81	(b)		
32	(c)	82	(c)		
33	(a)	83	(c)		
34	(d)	84	(b)		
35	(b)	85	(b)		
36	(c)	86	(c)		
37	(c)	87	(c)		
38	(d)	88	(b)		

39	(a)	89	(b)
40	(c)	90	(c)
41	(c)	91	(a)
42	(c)	92	(b)
43	(c)	93	(b)
44	(c)	94	(c)
45	(b)	95	(d)
46	(d)	96	(c)
47	(c)	97	(c)
48	(a)	98	(b)
49	(c)	99	(d)
50	(a)	100	(b)